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Maximization of equilibrium utility vs minimization of resources in the urban equilibrium[☆]

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ABSTRACT

We study welfare and the role of absentee landlords' rent capture in one of the building blocks of urban economics: the monocentric city model. If only a fraction (but not all) of the absentee landlords' land rents is captured and then redistributed, the market outcome minimizes resource usage but does not maximize residents' utility. Therefore, when the utility-maximizing planner is unable to implement full taxation of land rents, she would prefer to intervene in the market outcome, while a planner minimizing resources would not. This difference implies that policy prescriptions crucially depend on the chosen welfare function (utility vs resources) and the land rent taxation rate.

1. Introduction

The monocentric city model has been a fundamental building block in urban economics. It has been widely used to analyze the effects of transportation infrastructure, taxes, subsidies, and zoning regulations. Table A.1 contains references showing how pervasive the monocentric model has been. To enhance its intelligibility, land is assumed to be owned by absentee landlords who spend their rents elsewhere. However, whether the rents leave the economy depends on the planner's ability to tax absentee landlords. Usually, the planner is either assumed to be unable to capture or fully capture land rents.

Our paper deals with a seemingly unnoticed inconsistency in the urban economics literature. As also indicated in Table A.1, the monocentric model has been employed under two different approaches. One uses the equilibrium utility reached by identical households as a welfare measure and assumes that all land rents are captured. Under this framework, the literature shows that the market outcome maximizes equilibrium utility (e.g., Kanemoto, 1980). The second approach minimizes resources to achieve a given equilibrium utility, where resources refer to the inputs used to produce the composite good, commuting cost, and the opportunity cost of using land for housing rather than agriculture. But, this approach assumes that all land rents accrue to absentee landlords. The literature shows that the market equilibrium minimizes resource usage in this case (Fujita and Thisse, 2013).

Based on these facts, some authors have asserted that a planner who cares about equilibrium utility will mimic the market outcome because it minimizes resource usage (e.g., Duranton and Puga, 2015). However, this does not follow from the above results because land rent capture differs in each setting. We bridge the gap between the two approaches. We follow (Papageorgiou and Pines, 1999) and assume that the government captures a fraction of the land rents, which are then distributed lump-sum to all residents, to establish our main result:

- If only a fraction (but not all) of the absentee landlords' land rents is captured and redistributed, the market outcome minimizes resource usage but does not maximize residents' utility.

Therefore, when the utility-maximizing planner cannot fully tax land rents, she would prefer to intervene in the market outcome, while a planner minimizing resources would not. This difference implies that policy prescriptions crucially depend on the welfare function chosen (utility vs. resources) and the given land rent taxation rate, which, in all real situations, is way below 100%.

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2. A general monocentric model for land rents distribution

Alonso (1964), Mills (1967), and Muth (1969) developed the starting point of the modern urban economics literature: the monocentric city model. In that model, there is a central business district (CBD) where production takes place and accommodates all jobs. Modern references for a presentation include (Brueckner, 1987) and Duranton and Puga (2015).

Consider N identical households that choose a location along a closed linear city parameterized by $x \in [0, \overline{x}]$, where the CBD and the endogenous city boundary are located at $x = 0$ and $x = \overline{x}$ respectively. The land is owned by absentee landlords who do not demand housing and are taxed by the planner. Following (Papageorgiou and Pines, 1999), the government captures a fraction $\mu \in [0, 1]$ of the excess land rents, which are then distributed lump-sum equally to all residents. Residents take this transfer, denoted by E, as exogenous.

Workers earn an income y on top of the government transfer and face a constant commuting cost per unit of distance, t.[1] Households maximize their utility, represented by a quasi-concave function $u(c, q)$ that depends on the consumption of a composite good c and lot size of housing q. The price of c is normalized to one irrespective of location, while the per-unit land price is denoted by $p(x)$ as it varies with location; the main trade-off is between commuting distance – through the choice of location – and lot size. Households solve:

$$\begin{aligned} \max_{c,q,x} \quad & u(c, q) \\ \text{s.t.} \quad & y + E = tx + c + p(x)q \end{aligned} \tag{1}$$

Solving for c from the budget constraint, Eq. (1) can be rewritten as:

$$\max_{q,x} \; u\left(y + E - tx - p(x)q, q\right) \tag{2}$$

And the first-order condition for q is:

$$\frac{\partial u\left(y + E - tx - p(x)q, q\right)}{\partial q} \Big/ \frac{\partial u\left(y + E - tx - p(x)q, q\right)}{\partial c} = p(x) \tag{3}$$

The second equilibrium condition is that no one can improve their situation by unilaterally changing location. Consequently, the households' utility must be constant along the city:

$$u(y + E - tx - p(x)q(x), q(x)) = \overline{u} \quad 0 \le x \le \overline{x} \tag{4}$$

Conditions (3) and (4) allow for finding the price gradient, $p(x)$, and the equilibrium demand for land, $q(x)$, while $c(x)$ can be recovered from the budget constraint.

The urban market equilibrium is complete with four additional conditions: urban land rent in the city boundary equals the agricultural land rent r_a, the population N must fit inside the city, and the definitions of land rents and transfers are consistent:

$$p(\overline{x}) = r_a \tag{5}$$

$$N = \int_0^{\overline{x}} \frac{1}{q(x)} dx \tag{6}$$

$$R = \int_0^{\overline{x}} \left(p(x) - r_a\right) dx \tag{7}$$

$$E = \frac{\mu R}{N} \tag{8}$$

where the dependency on the parameters has been omitted for simplicity. Consequently, the budget constraint, along with Eqs. (3)–(8) fully characterize the spatial equilibrium (see, e.g., Brueckner, 1987).

For a given fixed income y, per-kilometer commuting cost t, population N, and share of land rent capture μ, the equilibrium is denoted by: $p^*(x, y, t, N, \mu)$, $c^*(x, y, t, N, \mu)$, $q^*(x, y, t, N, \mu)$, $\overline{x}^*(y, t, N, \mu)$ and $\overline{u}^*(y, t, N, \mu)$. The equilibrium of the model above is unique and Pareto efficient (Fujita, 1989; Papageorgiou and Pines, 1999).

[1] We do not include a time dimension in which residents allocate their time between work, commuting, and leisure. Income, y, is therefore exogenous.

We close this section by formalizing an efficiency result of Papageorgiou and Pines (1999). For all possible land rent capture structures, i.e., $\forall \mu \in [0, 1]$, the market equilibrium minimizes the resources – defined as the sum of total non-land consumption, total commuting cost, and the opportunity cost of the urban land – needed to achieve the equilibrium utility level $\overline{u}^*$.

Proposition 1. *Let $C^*(q(x), \overline{u}^*)$ be the quantity of the numeraire good for which $u(C^*(q(x), \overline{u}^*), q(x)) = \overline{u}^*$. Then, the market outcome coincides with the following minimization problem's solution:*

$$(P_{\min_\mu}) \qquad \begin{aligned} \min_{q(x),\overline{x}} \quad & \left(\int_0^{\overline{x}} \left[\frac{tx + C^*(q(x), \overline{u}^*)}{q(x)} + r_a \right] dx \right) \\ \text{s.t.} \quad & N = \int_0^{\overline{x}} \frac{1}{q(x)} dx \end{aligned}$$

Proof. See Papageorgiou and Pines (1999), chapter 4.4. □

Note that the result holds when the utility is constant along the city, which implies that the approach is not comparable with the first-best allocation of a Utilitarian planner. It is well-known that at the Utilitarian optimum, identical households at different locations have different utility levels (Mirrlees, 1972). The same holds when only a fraction of land rents are captured.

3. Equilibrium, utility maximization and minimization of resources

This section establishes the relationship between maximizing equilibrium utility and minimizing resources. First, it is helpful to recall the Herbert-Stevens (HS) framework (Herbert and Stevens, 1960) to discuss optimality. Consider the surplus of the economy defined as total income minus expenditure as in Fujita (1989)

$$N \cdot Y - \int_0^{\overline{x}} \left[\frac{tx + c(x)}{q(x)} + r_a \right] dx.$$

where $Y = y + E$ is the total per-capita income.

When Y includes the total differential land rents, the solution of maximizing surplus coincides with the minimization of resources. However, when a share of land rents is not captured by the planner and leaves the urban economy, $\mu < 1$ in our context, the surplus that matters is local.

The following Proposition establishes the relationship between maximizing equilibrium utility and minimizing resources:

Proposition 2. *Consider the problem of maximizing equilibrium utility for a given μ:*

$$(P_{\max_\mu}) \qquad \begin{aligned} \max_{c(x),q(x)} \quad & \overline{u} \\ \text{s.t.} \quad & u(c(x), q(x)) = \overline{u}, && 0 \le x \le \overline{x} \\ & N = \int_0^{\overline{x}} \frac{1}{q(x)} dx \\ & \frac{\mu R}{N} + y - tx - c(x) - p(x)q(x) = 0 && 0 \le x \le \overline{x} \\ & R = \int_0^{\overline{x}} \left[p(x) - r_a\right] dx \\ & p(\overline{x}) = r_a \end{aligned}$$

The problem $(P_{\max_\mu})$ is equivalent to the minimization of an alternative resource function that includes the rents that accrue to absentee landlords:

$$(\tilde{P}_{\min_\mu}) \qquad \begin{aligned} \min_{c(x),q(x)} \quad & \left(\int_0^{\overline{x}} \left[\frac{tx + c(x)}{q(x)} + r_a + (1 - \mu)(p(x) - r_a) \right] dx \right) \\ \text{s.t.} \quad & u(c(x), q(x)) = \overline{u}, && 0 \le x \le \overline{x} \\ & N = \int_0^{\overline{x}} \frac{1}{q(x)} dx \end{aligned}$$

$$R = \int_0^{\bar{x}} \left[p(x) - r_a\right] dx$$

$$p(\bar{x}) = r_a$$

Proof. Appendix A. □

Propositions 1 and 2 together establish the paper's main result: without full land rent capture and redistribution, the market equilibrium, which minimizes resource usage, does not maximize the equilibrium utility. In particular, when any fraction of land rents accrues to absentee landlords, minimizing resources coincides with the market equilibrium and not with the allocation that a Rawlsian planner would choose, **conditional** on that fraction.

The difference between the market outcome and the maximization of utility can be understood by comparing both minimization problems ($P_{\min_\mu}$) and ($\tilde{P}_{\min_\mu}$). The resources minimized in equilibrium are the commuting costs, the composite good costs and the agricultural rent. However, as long as some share of the differential land rent leaves the local economy, the cost of land is the agricultural rent plus the share of the differential that leaves the economy $(1-\mu)\cdot(p(x)-r_a)$. The minimization of these costs maximizes utility, as seen in Proposition 2. Therefore, only when all land rents are captured and redistributed ($\mu = 1$) is minimizing resources an appropriate welfare measure due to its equivalence with equilibrium utility maximization.

4. Conclusion

We have shown that maximizing equilibrium utility and minimizing resource usage are only equivalent when the planner fully captures and redistributes land rents. As an example, if the land rent tax is below 1, minimizing resources would imply that the market city has the optimal length, while if maximizing utility, the market city is too short.[2]

Note that our result is not only technical. Indeed, it implies that most welfare results found using the monocentric city model might change if a different welfare function and assumption about the land rents are used. This implication is not desirable, especially considering that the literature is highly fragmented, with many papers using either welfare function discussed in this paper.

Because of the nature of our study, we have not discussed the social desirability of increasing land rent capture. Intuitively, increasing the share of captured land rents would increase the equilibrium utility. Also, we do not extend our analysis to the case of a Utilitarian planner as in Mirrlees (1972) because we focus on urban equilibria that cannot be achieved there.

Data availability

No data was used for the research described in the article.

Appendix A. Proof of Proposition 2

Using the definition of the total population N, the problem can be restated as:

$$(P_{\max_\mu}) \qquad \max_{c(x),q(x)} \int_0^{\bar{x}} \frac{\bar{u}}{q(x)} dx$$

$$\text{s.t.} \quad u(c(x),q(x)) = \bar{u}, \qquad 0 \le x \le \bar{x} \tag{A.1a}$$

$$N = \int_0^{\bar{x}} \frac{1}{q(x)} dx \tag{A.1b}$$

$$\frac{\mu R}{N} + y - tx - c(x) - p(x)q(x) = 0 \qquad 0 \le x \le \bar{x} \tag{A.1c}$$

$$R = \int_0^{\bar{x}} \left[p(x) - r_a\right] dx \tag{A.1d}$$

$$p(\bar{x}) = r_a \tag{A.1e}$$

where the objective function is now $N \cdot \bar{u}$ instead of $\bar{u}$.

Let M be a household's total budget:

$$M = y + \frac{\mu R}{N} \tag{A.2}$$

$$= t(x) + c(x) + p(x)q(x) \tag{A.3}$$

Using (A.3) we obtain

$$p(x) = \frac{M - t(x) - c(x)}{q(x)} \tag{A.4}$$

Then, using the definition of R in (A.2), we get

$$M = y + \frac{\mu}{N}\int_0^{\bar{x}} p(x) - r_a \, dx$$
$$= y + \frac{1}{N}\int_0^{\bar{x}} p(x) - \mu r_a - (1-\mu)p(x) \, dx$$

Then, using (A.4)

$$M = y + \frac{1}{N}\int_0^{\bar{x}} \frac{M - t(x) - c(x)}{q(x)} - \mu r_a - (1-\mu)p(x) \, dx$$
$$= y + M + \frac{1}{N}\int_0^{\bar{x}} -\frac{t(x)+c(x)}{q(x)} - r_a - (1-\mu)\frac{u_q}{u_c} \, dx$$

Where we used the individual utility maximization condition $p(x) = \frac{u_q(c(x),q(x))}{u_c c(x),q(x))}$. The budget constraint (A.1c) can be replaced by

$$\int_0^{\bar{x}} \left[\frac{t(x)+c(x)}{q(x)} + \mu r_a + (1-\mu)\frac{u_q}{u_c}\right] dx = Ny \tag{A.5}$$

Now, the Lagrangian can then be written as

$$\Lambda_1(x) = \lambda_1 \int_0^{\bar{x}} \frac{\bar{u}}{q(x)} dx + \int_0^{\bar{x}} v(x)(u(c(x),q(x)) - \bar{u})dx + \delta\left[Ny - \int_0^{\bar{x}} \frac{t(x)+c(x)}{q(x)} + \mu r_a + (1-\mu)\frac{u_q}{u_c}\, dx\right] + \gamma\left[N - \int_0^{\bar{x}} \frac{1}{q(x)}\, dx\right] \tag{A.6}$$

where $v(x)$, δ and γ are the Lagrange multipliers associated with (A.1a), (A.5) and (A.1b), respectively. Applying the maximum principle (e.g., see Kanemoto, 1980), the first order conditions are:

$$v(x)u_c - \frac{\delta}{q(x)} - \delta\left(\frac{1-\mu}{u_c^2}\right)(u_{cq}u_c - u_q u_{cc}) = 0 \qquad 0 \le x \le \bar{x} \tag{A.7}$$

$$v(x)u_q - \frac{\lambda_1\bar{u} - \delta t(x) - \delta c(x) - \gamma}{q(x)^2} - \delta\left(\frac{1-\mu}{u_c^2}\right)(u_{qq}u_c - u_q u_{cq}) = 0 \qquad 0 \le x \le \bar{x} \tag{A.8}$$

$$\int_0^{\bar{x}} \frac{1}{q(x)} - \int_0^{\bar{x}} v(x) = 0 \tag{A.9}$$

$$\frac{\lambda_1\bar{u}}{q(\bar{x})} - \delta\left(\frac{t(\bar{x})+c(\bar{x})}{q(\bar{x})} + r_a\right) - \frac{\gamma}{q(\bar{x})} = 0 \tag{A.10}$$

Now, note that the objective function of the problem ($\tilde{P}_{\min}$) can be written as

$$\int_0^{\bar{x}} \frac{t(x)+c(x)}{q(x)} + r_a \, dx + (1-\mu)R = \int_0^{\bar{x}} \frac{t(x)+c(x)}{q(x)} + \mu r_a + (1-\mu)\frac{u_q}{u_c}\, dx \tag{A.11}$$

Then, the problem becomes

$$(\tilde{P}_{\min_\mu}) \qquad \min_{c(x),q(x)} \int_0^{\bar{x}} \frac{t(x)+c(x)}{q(x)} + \mu r_a + (1-\mu)\frac{u_q}{u_c}\, dx$$

[2] Intuitively, a more extended city would transfer landlord rents to the residents in the form of smaller aggregated housing prices. We deal with different ways to decentralize such an outcome in a separate paper, including the case with externalities.

$$\text{s.t.} \quad u(c(x), q(x)) = \bar{u}, \qquad 0 \le x \le \bar{x} \tag{A.12a}$$

$$N = \int_0^{\bar{x}} \frac{1}{q(x)} dx \tag{A.12b}$$

$$R = \int_0^{\bar{x}} p(x) - r_a \, dx \tag{A.12c}$$

$$p(\bar{x}) = r_a \tag{A.12d}$$

the Lagrangian is

$$\Lambda_2(x) = -\lambda_2 \int_0^{\bar{x}} \frac{t(x) + c(x)}{q(x)} + \mu r_a + (1-\mu)\frac{u_q}{u_c} \, dx + \int_0^{\bar{x}} \kappa(x)(u(c(x), q(x)) - \bar{u})dx + \rho \left[N - \int_0^{\bar{x}} \frac{1}{q(x)} \, dx \right] \tag{A.13}$$

where $\kappa(x)$ and ρ are the Lagrange multipliers associated with (A.12a) and (A.12b). The first-order conditions are

$$\kappa(x)u_c - \frac{\lambda_2}{q(x)} - \lambda_2 \left(\frac{1-\mu}{u_c^2} \right) (u_{cq}u_c - u_q u_{cc}) = 0 \qquad 0 \le x \le \bar{x} \tag{A.14}$$

$$\kappa(x)u_q - \frac{-\lambda_2 t(x) - \lambda_2 c(x) - \rho}{q(x)^2} - \lambda_2 \left(\frac{1-\mu}{u_c^2} \right) (u_{qq}u_c - u_q u_{cq}) = 0 \qquad 0 \le x \le \bar{x} \tag{A.15}$$

$$\int_0^{\bar{x}} \frac{1}{q(x)} - \int_0^{\bar{x}} \kappa(x) = 0 \tag{A.16}$$

$$-\lambda_2 \left(\frac{t(\bar{x}) + c(\bar{x})}{q(\bar{x})} + r_a \right) - \frac{\rho}{q(\bar{x})} = 0 \tag{A.17}$$

Finally, suppose that for some feasible control variables and parameters $(c^*(x), q^*(x), \bar{x}^*, \bar{u}^*)$ there exists a set of multipliers $(\nu^*(x), \lambda_1^*, \delta^*, \gamma^*)$ such that (A.7)–(A.10) hold. Then, note that there also exists a set of multipliers $(\kappa^*(x), \lambda_2^*, \rho^*)$ such that (A.14)–(A.17) also hold for $(c^*(x), q^*(x), \bar{x}^*, \bar{u}^*)$. Indeed, consider

$$(\kappa^*(x), \lambda_2^*, \rho^*) = (\nu^*(x), \delta^*, \gamma^* - \lambda_1^* \bar{u}) \tag{A.18}$$

Conversely, suppose that for some feasible control variables and parameters $(c^*(x), q^*(x), \bar{x}^*, \bar{u}^*)$ there exists a set of multipliers $(\kappa^*(x), \lambda_2^*, \rho^*)$ such that (A.14)–(A.17) hold. Then, note that there also exists a set of multipliers $(\nu^*(x), \lambda_1^*, \delta^*, \gamma^*)$ such that (A.7)–(A.10) also hold for $(c^*(x), q^*(x), \bar{x}^*, \bar{u}^*)$. Indeed, consider

$$(\nu^*(x), \lambda_1^*, \delta^*, \gamma^*) = (\kappa^*(x), m, \lambda_2^*, \rho^* + m\bar{u}) \quad \text{for any} \quad m \ge 0 \tag{A.19}$$

Therefore, problems $(P_{\max_\mu})$ and $(\tilde{P}_{\min_\mu})$ are equivalent.

Appendix B. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.econlet.2024.111772.

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