Transportation Research Part B 176 (2023) 102803

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

Fare evasion in public transport: How does it affect the optimal design and pricing?

Raúl Ramos [a,*], Hugo E. Silva [a,b,c,d,1]

[a] Departamento de Ingeniería de Transporte y Logística, Pontificia Universidad Católica de Chile, Santiago, Chile
[b] Instituto de Economía, Pontificia Universidad Católica de Chile, Santiago, Chile
[c] Centro de Desarrollo Urbano Sustentable, CEDEUS, Chile
[d] Instituto Sistemas Complejos de Ingeniería (ISCI), Chile

ARTICLE INFO

Keywords:
Public transport
Fare evasion
Fare dodging
Pricing
Design
Subsidy
Inspection
Welfare

ABSTRACT

Fare evasion produces significant revenue losses in public transport systems. Recent research has found that low service quality and high prices are important determinants of fare evasion. However, the economic literature that studies the optimal public transport provision has overlooked the phenomenon. We develop a demand model of horizontal differentiation to investigate how fare evasion affects the design and pricing of public transport that maximizes utilitarian social welfare. We show analytically that fare evasion can create incentives to reduce public transport prices and improve service quality, putting upward pressure on its subsidization. We perform numerical simulations and sensitivity analysis to quantify the impact of these incentives on the public transport provision. These simulations confirm that the incentive to reduce fare evasion can lead to an optimal design and price that requires subsidies.

1. Introduction

Fare evasion in public transport is a major concern for governments worldwide because it produces millions of dollars in lost revenues that risk the systems' financial health (Bonfanti and Wagenknecht, 2010). For example, 30% of bus users in Santiago, Chile, and Bogota, Colombia, do not pay the fare, producing annual losses estimated at USD 140 million (Porath and Galilea, 2020; Probogotá, 2022). On the other hand, evasion rates are usually below 5% in the developed world. However, some independent studies suggest that official reports may be underestimating the problem. For example, recent studies have found double-digit evasion rates in cities like Cagliari and Reggio Emilia, Italy (Barabino et al., 2022; Bucciol et al., 2013), Melbourne (Currie and Delbosc, 2017), Lyon (Egu and Bonnel, 2020), and Athens (Milioti et al., 2020).

In line with the increasing concern of transport authorities, the number of studies focused on understanding why people evade payment has grown exponentially in recent years (see, Barabino et al., 2020, for a recent review). These studies show that fare evasion depends on factors such as the users' characteristics and attitudes, the deterrence and enforcement measures implemented by authorities, and the contractual incentives to public transport operators and drivers (Dai et al., 2018; Clarke et al., 2010; Ramírez et al., 2022; Delbosc and Currie, 2019; Tamblay et al., 2017). Perhaps more significant for public transport authorities is that low service quality and high prices are essential determinants of fare dodging (Busco et al., 2021; Barabino et al., 2015; Guzman et al.,

* Correspondence to: Pontificia Universidad Católica de Chile, Avda. Vicuña Mackenna 4860, Macul, Santiago, Chile.
E-mail addresses: rramos4@uc.cl (R. Ramos), husilva@uc.cl (H.E. Silva).
1 Pontificia Universidad Católica de Chile - Avda. Vicuña Mackenna 4860, Macul - Santiago - Chile.

https://doi.org/10.1016/j.trb.2023.102803
Received 24 January 2023; Received in revised form 27 July 2023; Accepted 30 July 2023

2021; Cools et al., 2018). This occurs for two reasons: First, evaders are usually dissatisfied with the price and overall level of service (Delbosc and Currie, 2016a,b; González et al., 2019). Second, conditions like crowding at bus stops and vehicles favor evasion because they increase the anonymity perception and the so-called contagion effect, which makes dodging easier (Cantillo et al., 2022; Currie and Delbosc, 2017; Salis et al., 2018).[2] Consequently, recent studies have recommended increasing the level of service and the subsidies to reduce the number of evaders in public transport (see, e.g., Porath and Galilea, 2020; Barabino et al., 2022; Milioti et al., 2020). Nevertheless, the economic efficiency of this proposal and its consequences for operators' and users' surplus have not been studied.

The transport economic literature has investigated in detail the welfare-maximizing public transport provision (see Hörcher and Tirachini, 2021, for a recent review). However, this strand of the literature disregards evasion or considers it an exogenous variable. On the other hand, the economic literature on fare evasion has focused on inspection and fines, overlooking the effect of quality and prices on the evading behavior (see, e.g., Barabino and Salis, 2019). This paper aims to fill these gaps by acknowledging that fare dodging is partially determined by the decisions of transport authorities regarding public transport quality, price, and inspection rate. Thus, our objective is to investigate the effect of fare evasion on the welfare-maximizing design and pricing of public transport.

We develop a theoretical model where a mass of potential users chooses between paying or evading when using an isolated bus line provided by a public operator. Individuals base their decision on the service quality, fare, and inspection rate of public transport. Analytical expressions show that fare evasion creates incentives to reduce public transport fares below the standard marginal-cost pricing, putting upward pressure on subsidies. Regarding service quality, evasion creates an incentive to improve the frequency if reducing the inspection rate raises inspection profits, which may occur if the inspection is too costly or if revenues from fines are negligible.

We illustrate our theoretical model with numerical simulations aiming to reflect a representative public transport system with a 30% evasion rate. Our results show that the evasion rate in this baseline scenario is ten percentage points above optimal, so the social planner increases inspection to reduce evasion and raise revenues from fines. Consequently, the cost recovery ratio rises from 60% in the baseline scenario to almost 80% in the optimal solution. Nevertheless, unlike the previous literature, the optimal design and pricing of public transport considering fare evasion requires subsidies for realistic values of the cost of public funds (Basso and Silva, 2014; Börjesson et al., 2019; Hörcher et al., 2020). This confirms that the Mohring effect (1972, 1976) and the incentive to reduce fare evasion can countervail the distortions in the markets where the government collects taxes, showing that the relationship between fare evasion and the price and quality of public transport strongly affects its optimal provision.

The rest of the paper is structured as follows. Section 2 reviews the related literature. Section 3 presents the model. Section 4 describes and interprets the analytical solution. Section 5 shows a numerical implementation of the model, and Section 6 concludes.

2. Related literature

This section summarizes the three relevant strands of literature to understand our contribution. First, we review the empirical studies that relate fare evasion with public transport quality and price. Second, we analyze the economic models of fare evasion. Third, we review the literature on welfare-maximizing public transport design and pricing.

The empirical evidence that relates fare evasion with public transport quality and price comes from studies conducted in Latin America, Italy, and Australia. These studies find that fare dodging increases with low frequency and reliability (Busco et al., 2021; Allen et al., 2019; Guarda et al., 2016b), crowding (Cantillo et al., 2022; Guarda et al., 2016a), users' dissatisfaction (Guzman et al., 2021; Barabino et al., 2015), and high prices (Cools et al., 2018; Troncoso and de Grange, 2017; Porath and Galilea, 2020). Other papers seek to understand evasion from the users' perspective and link these results with the intentionality of evaders in what has been called the *spectrum of fare evasion* (Delbosc and Currie, 2019).

Users that deliberately evade payment (i.e., those who perform acts such as jumping ticketing barriers) are usually dissatisfied with the overall service quality and the price of public transport (Guzman et al., 2021; González et al., 2019; Delbosc and Currie, 2016a; Suquet, 2010; Delbosc and Currie, 2016b). Unintentional and opportunistic evaders are also dissatisfied with the service quality. However, they only evade under some circumstances: (i) when buses are crowded because it is more difficult to reach the ticket validation machine, (ii) when it is easier to board by the rear door, where there is less control over who pays, and (iii) when large groups board at each bus stop because it increases the perception of anonymity and the so-called contagion effect (Guarda et al., 2016a,b; Porath and Galilea, 2020; Allen et al., 2019; Laboratiorio de Innovacion Pública, 2018; Lee, 2011; Salis et al., 2018; Currie and Delbosc, 2017; Delbosc and Currie, 2016b). Based on these results, some authors have recommended increasing the public transport level of service and subsidization to reduce fare evasion in Santiago (Cantillo et al., 2022; Porath and Galilea, 2020; Guarda et al., 2016b), Cagliari (Barabino et al., 2022; Barabino and Salis, 2020), Bogota (Guzman et al., 2021), Athens (Milioti et al., 2020) and Flanders (Cools et al., 2018). However, the economic benefits and costs of implementing such a strategy have never been studied.

Another group of studies has proved that ticket inspection is an effective method for reducing fare evasion in public transport systems (see, e.g., Clarke et al., 2010; Killias et al., 2009; Guarda et al., 2016a). Consequently, most of the previous economic models focus on the inspection rate that maximizes the operator's profit by assuming that users make a rational decision between paying the fare and the perceived probability of being inspected and fined (Boyd, 1989; Kooreman, 1993; Barabino and Salis, 2019; Barabino et al., 2013, 2014; Sasaki, 2014; Guarda et al., 2016a).

[2] The contagion effect is that a user's exposure to fare evasion increases the probability of evading payment.

Two papers that depart from these models are Silva and Kahn (1993), and Buehler et al. (2017). The former studied a firm that provides public transport. With a homogeneous population, optimal provision leads to zero evasion; however, a positive number of evaders is optimal when the population is heterogeneous. Buehler et al. (2017) consider a monopoly that chooses the fine and the price, finding that fare evasion can be seen as a discrimination instrument that allows the firm to charge higher prices for non-evaders. On the other hand, if the firm maximizes social welfare, the price equals the expected value of the fine (i.e., the probability of being inspected times the fine).

More recently, Besfamille et al. (2022) studied the problem of a regulated monopoly that faces fare evasion. They focus on reputational costs rather than fines and allow users to choose between consuming (paying or evading) or leaving the market based on the good's quality and price. They find that the optimal price and effort to reduce fare evasion depend on the information available to the regulator and whether the product's quality is endogenous to the monopoly.

An important limitation of previous fare evasion models is that they do not include particular characteristics of the public transport market that are fundamental for the analysis. First, users spend a valuable resource when using public transport: their time (Jara-Díaz, 2007; Jara-Díaz and Gschwender, 2005; Small and Verhoef, 2007). Second, public transport usually exhibits increasing returns to scale in the sum of users and operating costs (Allport, 1981; Jara-Díaz et al., 2022; Fielbaum et al., 2020). Third, public transport usage generates externalities such as crowding and congestion (Yap et al., 2020; Bansal et al., 2019, 2022). The impact of these characteristics on the welfare-maximizing public transport provision has been broadly studied in the economic literature (see, e.g., Hörcher and Tirachini, 2021, for a recent review). In his seminal work, Mohring (1972, 1976) proved that an increase in the demand for public transport reduces the waiting time cost when it is dealt with an increase in frequency (Silva, 2021). Consequently, the optimal fare does not cover the operating cost and requires subsidies. This so-called Mohring effect, together with unpriced car externalities and distributional considerations, are the main arguments for public transport subsidization (Adler et al., 2021; Adler and van Ommeren, 2016; Börjesson et al., 2020; Matas et al., 2020).

Mohring's work has been extended in various directions. For example, some recent models have incorporated externalities such as crowding, congestion, and agglomeration economies (Tirachini et al., 2014; Hörcher et al., 2018; Tirachini and Hensher, 2011; Hörcher et al., 2020), while other authors focus on the spatial and temporal dimensions of the public transport provision (Jara-Díaz et al., 2020; Fielbaum et al., 2021). Other papers study the effects of budget restrictions and the fact that public funds have a non-negligible cost to society (Sun et al., 2016; Jara-Díaz and Gschwender, 2009; Ramos and Silva, 2022; Jara-Díaz et al., 2022). Furthermore, many studies have investigated the interaction of public transport subsidization with other modes and transport policies like car congestion pricing and dedicated bus lanes (Basso and Silva, 2014; Börjesson et al., 2017; Gómez-Lobo et al., 2022; Basso and Jara-Díaz, 2012). However, to the best of our knowledge, the public transport economic literature has disregarded fare evasion or considered it an exogenous variable.

In sum, how fare evasion affects welfare-maximizing public transport design and pricing remains a gap in the literature. First, most papers on evasion do not consider essential and unique characteristics of public transport provision. Second, the public transport economic literature that focuses on design and pricing has overlooked the fare dodging phenomenon. Thus, our paper contributes to filling this gap by proposing a tractable model to study the fare evasion problem in the context of optimal public transport provision.

3. The model

We consider a bus line that serves a mass of individuals distributed homogeneously along a corridor, as in the seminal works by Mohring (1972, 1976) and Jansson (1980, 1984). This simple and tractable approach allows us to obtain transparent insights into the impact of fare evasion in public transport provision and compare our results with the well-known pricing and quality rules obtained in previous studies. Following fare evasion literature (Buehler et al., 2017; Besfamille et al., 2022; Silva and Kahn, 1993), we model individual behavior as a discrete choice between (i) traveling by bus and paying the fare (formal users or non-evaders), (ii) traveling by bus evading the payment and assuming the risk of being inspected and fined (informal users or evaders), and (iii) not traveling (leave the market).

A public operator provides the bus service while a public agency performs the inspection activities. A social planner defines the bus line frequency f, the fare τ, and the probability π of being inspected and fined (i.e., the inspection rate). For the sake of simplicity, we assume that the bus capacity is fixed and sufficiently high for the bus occupancy to be below the maximum possible level at an interior solution.

3.1. Demand model

Let us consider a benefit function $B(Y^p, Y^e)$, where Y^p and Y^e denote the number of formal and informal users. Ignoring income effects, the benefit function represents the consumers' benefit as their total willingness to travel for a particular combination $\{Y^p, Y^e\}$ (see, e.g., Kraus, 2003; Pressman, 1970; Tirachini and Hensher, 2012). Therefore, it is a generalization of the area under the inverse demand function (Small and Verhoef, 2007, pp. 155–159). Then, the marginal benefit from traveling formally or informally, which can be interpreted as an inverse demand function, is given by:

$$B_i = \frac{\partial B(Y^p, Y^e)}{\partial Y^i},\ i \in \{p, e\}, \tag{1}$$

where subscripts denote derivatives. Just as with any inverse demand function, B_i decreases with the number of users of type Y^i. This means that the marginal valuation of making the trip decreases with both formal and informal demand; that is:

$$B_{ij} = \frac{\partial^2 B(Y^p, Y^e)}{\partial Y^i \partial Y^j} < 0,\ i, j \in \{p, e\}. \tag{2}$$

In equilibrium, the marginal willingness to pay B_p equals the so-called generalized price, which is the total cost incurred by a representative formal user (see, Small and Verhoef, 2007, pp 83-85). Therefore, the generalized price is computed as the fare τ plus the generalized cost $GC(Y,f)$, a function that values the waiting time, in-vehicle time, and crowding costs in monetary units:[3]

$$B_p = \tau + GC(Y,f). \tag{3}$$

Following the public transport literature, we let $GC(Y,f)$ be an increasing function of the total demand $Y = Y^p + Y^e$ because it raises travel times and crowding, and a decreasing function of the frequency because it reduces waiting times (i.e., $GC_Y > 0$ and $GC_f < 0$; where, again, subscripts denote first derivatives).[4]

Similarly, B_e equals the generalized price perceived by evaders. Of course, evaders do not incur any monetary cost; however, they may be inspected and fined with a certain probability, so they make decisions based on an Expected Value of the Fine (EVF), the generalized cost, and the personal evasion cost $EC(Y,f)$:

$$B_e = \pi \cdot h + GC(Y,f) + EC(Y,f). \tag{4}$$

Assuming risk-neutral users with perfect information about the probability of being inspected, the EVF is computed as the product of the inspection rate π and the value of the fine h. We also assume there is a personal evasion cost $EC(Y,f)$, which is a function that captures how service quality affects evading behavior in monetary units. As explained in Section 2, fare evaders usually excuse their behavior on the low service quality of public transport. Furthermore, crowding at bus stops, doors, and inside vehicles increase the anonymity perception and the so-called contagion effect, which makes dodging payment easier. The personal evasion cost $EC(Y,f)$ captures these effects; hence, it increases with frequency and decreases with the number of users (i.e., $EC_f > 0$, $EC_Y < 0$).

This benefit function specification admits typical demand systems such as Random Utility Models (RUM) and Constant Elasticity of Substitution (CES) models (Anderson et al., 1988, 1989). It follows the tradition of modeling horizontal or subjective differentiation in a reduced-form fashion and captures evaders' heterogeneity or the so-called *spectrum of fare evasion*.

For example, career evaders rarely pay the fare and take pride in their actions due to social or political considerations (Delbosc and Currie, 2016a). They believe that their behavior is acceptable to protest against the government, the public transport system, or private companies (Porath and Galilea, 2020; Busco et al., 2021). This means that they benefit much more from traveling informally than formally. Therefore, career evaders dodge even if the generalized price perceived when evading is higher than that perceived when paying. (i.e., when $\tau < \pi \cdot h + EC(Y,f)$). Analogously, the so-called never-evaders strongly believe that not paying the fare is wrong. They value traveling formally much more than informally, so they never evade payment. Finally, users with a similar valuation for traveling formally and informally will decide based on the fare, inspection rate, and service quality, which reflects the behavior of deliberate and opportunistic fare evaders.[5]

3.2. Social welfare

We assume a public operator that provides the necessary fleet for a given demand and frequency at a cost $OC(Y,f)$, an increasing function of the frequency and the total number of users (i.e., $OC_f > 0$, $OC_Y > 0$). The public operator collects the fare and receives a subsidy from the government if the revenues do not cover the operating cost. We also assume another public agency that performs the inspection activities at a cost $IC(\pi)$, an increasing function of the inspection rate.[6]

We know that fare evasion also depends on the collection technology. However, it is reasonable to assume that the collection method does not change over time for financial and practical reasons. For example, smart cards are the current fare collection technology in Bogota and Santiago. This is unlikely to change shortly because it would imply replacing or removing the ticketing machines in metro/BRT stations and inside buses and sometimes costly renegotiation of long-term contracts with service providers. We also acknowledge that other measures, such as off-board ticketing, entrance barriers, and CCTV, can deter evaders. However, worldwide experience has proven that inspection is the most effective method to control fare dodging (Killias et al., 2009; Clarke et al., 2010; Guarda et al., 2016a). Therefore, for the sake of simplicity, we follow the fare evasion literature and assume that inspection is the only anti-evasion policy under the control of transport authorities (see, e.g., Barabino and Salis, 2019).

We compute social welfare as the weighted sum of the consumers' surplus and the financial results of the public transport operator $\Pi^p = \tau \cdot Y^p - OC$ and the inspection agency $\Pi^e = \pi \cdot h \cdot Y^e - IC$; that is:

$$W = CS + [1+\lambda][\Pi^p + \Pi^e], \tag{5}$$

[3] The generalized cost function also includes the access time; however, since the bus stop locations are fixed in our model, we can normalize access time to zero.

[4] We acknowledge that increasing the frequency may raise the users' generalized cost under certain conditions; for example, when congestion dominates waiting time savings. We opt for following the conventional assumption that leads to increasing returns to scale in the user's costs to be consistent with the public transport literature. However, if increasing the frequency raises the users' generalized cost, our main results hold, although the Mohring effect may not be active (see Section 4.4).

[5] We acknowledge that our theoretical model does not explicitly consider the so-called accidental evaders. These individuals evade due to circumstances different from an economic calculation, for example, because they forget their smart card or run out of money unexpectedly. Therefore, their behavior does not depend on the price, inspection rate, or service quality. However, several studies show that they represent a small portion of fare evaders (see, e.g., González et al., 2019; Salis et al., 2018), so we can assume the number of accidental evaders is fixed.

[6] Early versions of our model considered a more complex representation of the inspection technology where it depends on the occupancy rate and the number of users. However, these effects do not add value to our main results' interpretation. Since our paper focuses on the quality and price of public transport, we opted for this simplified representation of inspection technology.

where the parameter $\lambda \geq 0$ measures the tax distortion or the so-called shadow cost of public funds. This monetary distortion is due to the collection of local taxes on income, capital, or consumption (Laffont and Tirole, 1993). Therefore, if the government spends \$1 subsidizing the bus service, society pays $\$[1+\lambda]$. Conversely, if the system's profit increases by \$1, the society earns $\$[1+\lambda]$. We use this straightforward formulation rather than modeling general and labor equilibrium effects because it is simpler and allows for comparing our results more directly with previous literature that also adopts this approach (see, e.g., Proost and van Dender, 2008; Basso and Silva, 2014; Börjesson et al., 2019; Hörcher et al., 2020).

The consumers' surplus is given by the difference between the benefit function and the total cost perceived by formal and informal users:

$$CS = B(Y^p, Y^e) - Y^p[\tau + GC(Y, f)] - Y^e[\pi \cdot h + GC(Y, f) + EC(Y, f)]. \tag{6}$$

Then, replacing Eq. (6) in Eq. (5), we can rewrite the social welfare function as:

$$W = B(Y^p, Y^e) + \lambda\left[\tau \cdot Y^p + \pi \cdot h \cdot Y^e\right] - Y \cdot GC(Y, f) - Y^e \cdot EC(Y, f) - [1+\lambda]\left[OC(Y, f) + IC(\pi)\right]. \tag{7}$$

The first term of Eq. (7) is the total benefit or the gross consumer surplus obtained by a given combination of formal and informal demand. The second term accounts for the social benefits from revenues. It has a positive effect on social welfare because the rent extracted from users reduces the need for distortionary taxes on other markets. Note, however, that if public funds are not costly (i.e., $\lambda = 0$), revenues from fares and fines are just transfers between the users and the government that do not affect social welfare. The third term is the total generalized cost perceived by users, while the fourth accounts for the total evasion cost perceived by informal users. Finally, the last term discounts the cost for society of public transport operation and inspection activities.

Replacing equilibrium conditions Eqs. (3) and (4), in the social welfare function Eq. (7), we can write the planner's welfare maximization problem as:[7]

$$\underset{Y^p, Y^e, f}{\text{Max}} \quad B(Y^p, Y^e) + \lambda\left[\sum_{i\in\{p,e\}} Y^i \cdot B_i\right] - [1+\lambda]\left[Y \cdot GC(Y, f) + Y^e \cdot EC(Y, f) + OC(Y, f) + IC\left(\frac{B_e - GC(Y, f) - EC(Y, f)}{h}\right)\right]. \tag{8}$$

From now on, we assume that the fine is an exogenous parameter instead of considering it a decision variable. We make this assumption because the objective function in Eq. (8) is strictly increasing with h; therefore, if we consider the fine as an endogenous variable, the optimal fine always equals its maximum possible value given by law (see Appendix A for the proof and Buehler et al., 2017). This occurs for two reasons: First, there is a direct effect on social welfare since increasing the fine raises the revenues from inspection, which in turn, reduces the need for distortionary taxes. However, note that this effect is only active when $\lambda > 0$, as seen in the second term Eq. of (7).

Second, since we assume risk-neutral users, the EVF increases with the value of the fine (see equilibrium condition Eq. (4)). Consequently, the planner can implement a given value of Y^e by increasing the fine and reducing the inspection rate as long as the product $\pi \cdot h$ remains constant. Therefore, the social planner will increase the fine until it reaches its maximum admissible value to save money on inspection. This effect is captured by the term $IC(\cdot)$ in Eq. (8) and is active whether public funds are costly or not.

Thus, h can be interpreted as the optimal fine, which is the maximum value given by law. Furthermore, since h is an exogenous parameter, we can normalize its value to one; therefore, from now on, the Expected Value of the Fine is given by π. This normalization does not affect our results because it just scales all the prices by the factor $1/h$. Moreover, if the fine is not normalized, it appears as a parameter that does not change the interpretation of the optimal pricing and frequency rules, as can be seen in Appendix A.

Finally, note that the welfare function defined in Eq. (7) includes the evaders' surplus, which may raise ethical questions since fare dodging is considered antisocial or illegal. However, this so-called utilitarian approach is standard in the literature on fare evasion (Buehler et al., 2017; Silva and Kahn, 1993; Besfamille et al., 2022), and other illicit activities, such as tax evasion (Pestieau and Possen, 1991; Trandel, 1992; Goerke, 2017) and software piracy (Banerjee, 2003; Waters, 2015; Bae and Choi, 2006; Yoon, 2002). We also follow the utilitarian approach to be consistent with previous works. Nevertheless, rather than commit to this position, we aim to investigate its consequences for the optimal design and pricing of public transport (see Sandmo, 1981, for a brief discussion).

4. Analytical solution

This section describes and interprets the first-order conditions of the planner's maximization problem to understand how fare evasion distorts the standard public transport optimal provision. We assume that social welfare is strictly concave, so the first-order conditions characterize a unique interior solution for the optimal fare, inspection rate, and frequency. First, we analyze the welfare-maximizing fare and inspection rate when public funds are not costly, which is the most common assumption in the public transport literature. Second, we study how the cost of public funds impacts the optimal pricing and inspection rules. Third, we interpret the optimal frequency rule, and finally, we analyze how fare evasion affects optimal public transport subsidization.

[7] Note that the last term of Eq. (8) is the sum of users, operating, and inspection costs, or equivalently, the Value of Resources Consumed (VRC). Therefore, if we consider quantities Y^i as parameters, the optimal frequency can be obtained by minimizing VRC, which is the standard approach in the public transport literature (Jara-Díaz and Gschwender, 2003).

4.1. Optimal price and inspection rate without tax distortions

The welfare-maximizing fare when public funds are not costly or when the pricing of public transport does not need to be adjusted for other tax distortions (i.e., $\lambda = 0$), is given by:

$$\tau^* = \left[OC_Y + Y^p \cdot GC_Y\right] + Y^e[GC_Y + EC_Y] + IC_p. \tag{9}$$

where IC_p is the inspection cost savings due to a marginal increase in formal demand; that is:

$$IC_p = IC_\pi \left[B_{ep} - GC_Y - EC_Y\right] \leq 0. \tag{10}$$

Note that Eq. (9) is not a closed-form expression because the right-hand side depends on the demand and frequency. However, we can interpret the components of τ^* conditional on these variables.

The first term in square brackets is the sum of the marginal operating cost and the marginal external cost imposed on formal users. It is the welfare-maximizing public transport fare in a world without evasion; therefore, the sign of the remaining terms of Eq. (9) will indicate how fare evasion distorts the standard marginal-cost pricing rule.

The second term on the right-hand side of Eq. (9) is the marginal external cost that one additional user imposes on evaders. It has two components: the increase in the generalized cost due to the rise in crowding and travel times, and the reduction in the personal evasion cost because crowding at stops and buses raises the anonymity perception and makes evading easier. The empirical evidence suggests that this term is negative because increasing the total demand increases fare evasion (Salis et al., 2018; Currie and Delbosc, 2017; Porath and Galilea, 2020; Guarda et al., 2016b). For example, in Santiago, a 1.0% increase in boardings and bus occupancy raises the number of evaders by 1.1% and 0.8%, respectively (Guarda et al., 2016a). This indicates that one additional user reduces the cost perceived by evaders. Hence, the social planner discounts this effect to the optimal fare.

Finally, the third term is the inspection cost savings due to a marginal increase in formal demand. Under the standard assumption that the informal demand increases with public transport prices (i.e., $B_{ep} - GC_Y - EC_Y < 0$), the government saves money on inspection by reducing the fare. Therefore, these savings are discounted from the public transport optimal price. Note that the inspection savings increase with the marginal inspection cost; hence, the more costly the inspection is, the lower the optimal fare. Moreover, the greater the degree of substitution between paying and evading, the more significant the inspection savings are when the price is reduced. Thus, τ^* decreases with the substitutability between formal and informal demand.

Then, we can summarize the effect of fare evasion on the public transport optimal price when public funds are not costly, as follows: *If the informal demand for public transport increases with its price and total number of users, fare evasion distorts the optimal public transport pricing downwards.*

This suggests that authorities may have strong incentives to reduce public transport fares below the standard marginal-cost pricing rule; and, therefore, increase public transport subsidization. Note, however, that if the fare of a public transport system is away from the first-best value due to financial or political constraints, the optimal response could be to reduce subsidization. This means that the intuition above holds when the optimal fare is compared to the marginal-cost pricing. We come back to this issue in the numerical analysis of Section 5.

Eq. (9) also reveals that the welfare-maximizing inspection rate π^* plays a vital role in the design and pricing of public transport. Recalling that the Expected Value of the Fine (EVF) equals π, we compute π^* from the first-order conditions as:

$$\pi^* = \tau^* - IC_\pi \left[1 - \frac{B_{ep}}{B_{ee}}\right] \left[-B_{ee}\right]. \tag{11}$$

The relevant difference between τ^* and π^* is their impact on the inspection costs: reducing π impacts the inspection costs directly, an effect captured by B_{ee}. On the other hand, the effect of reducing τ depends on the degree of substitution between paying and evading, which is captured by B_{ep}. To illustrate this, let us analyze some particular cases.[8] If paying and evading are perfect substitutes, $B_{ee} = B_{ep} = d'$, and $\tau^* = \pi^*$ (where d' is the derivative of the inverse demand function for the total demand). This occurs because reducing the fare has the same effect on the inspection costs as increasing the inspection rate. Furthermore, note from Eqs (9) and (11) that a perfectly inelastic total demand (i.e., $d' \to -\infty$) leads to the corner solution $\tau^* = \pi^* = 0$ because, in this case, the optimal price is the one that minimizes the inspection costs. If paying and evading are imperfect substitutes $B_{ep}/B_{ee} < 1$; thus, $\tau^* > \pi^*$. Moreover, the lower the degree of substitution between paying and evading, the higher the difference between τ^* and π^*, reaching its maximum value when paying and evading are independent (i.e., $B_{ep} = 0$).

Two interesting properties of the optimal pricing and inspection rules deserve some discussion. First, the positive externality $Y^e \cdot EC_Y$ is discounted from the optimal fare and inspection rate. In other words, optimal pricing accounts for an externality that makes evasion easier. Second, if the marginal inspection cost is zero (i.e., $IC_\pi = 0$), the optimal EVF equals the optimal fare. Thus, even if the inspection is costless, the optimal level of evasion is positive.

Policymakers may question these results because they suggest that some level of fare evasion is socially acceptable. One may argue that, due to its unethical nature, the optimal level of evasion must be zero. However, the positive evasion rate arises naturally from our utilitarian social welfare function and is in line with previous works on tax and fare evasion and software piracy (see, e.g., Waters, 2015; Pestieau and Possen, 1991; Silva and Kahn, 1993). The intuition is: if all users pay the fare, those who intend to evade payment would be worse off or would not use the bus. Thus, from a utilitarian perspective, not having evaders may decrease social welfare, so the social planner allows those with a greater valuation for traveling informally to dodge payment. We will return to this issue in the concluding section.

[8] See Small and Verhoef (2007), pp 157 and 233; for a detailed explanation.

4.2. Optimal price and inspection rate with costly public funds

The results in the previous section rely on the assumption that public funds do not have any cost to society. However, public funds have a non-negligible cost due to the distortion in the markets where the government collects taxes. Thus, this section analyzes how the shadow cost of public funds impacts the welfare-maximizing pricing rule.

The optimal fare and inspection rate when public funds are costly (i.e., $\lambda > 0$), denoted by ($\sim$), are given by the following Ramsey formula for interdependent demand:

$$\xi_p \left[\frac{\tilde{\tau} - \tau^*}{\tilde{\tau}}\right] = \xi_e \left[\frac{\tilde{\pi} - \pi^*}{\tilde{\pi}}\right] = \frac{\lambda}{1+\lambda}. \tag{12}$$

Where τ^* and π^* are given by Eqs (9) and (11) but evaluated at the optimal demand and frequency, while ξ_p and ξ_e are the formal and informal demand *superelasticities*.[9]

Note that this result is similar to the one obtained by Hörcher et al. (2020) when studying the effect of agglomeration economies in the public transport provision with a substitute mode. The key difference is that instead of two modes, we have two different types of consumption: formal and informal. However, the interpretation is similar: the optimal price and inspection rate are set above τ^* and π^* because revenues reduce the need for costly taxes. Moreover, the pricing and inspection rules increase with λ and depend on both the direct and cross elasticities because increasing the public transport price increases the operator's revenues but also raises revenues from fines, an effect captured by the demand *superelasticity*.

Eq. (12) shows a trade-off between the benefit of reducing the public transport price, captured by τ^*, and the cost to society in terms of distortionary taxes. Therefore, the value of the optimal fare depends on which of these two effects dominates. We can understand this trade-off by analyzing the particular cases when paying and evading are perfect substitutes, which results in the following pricing rule:

$$\tilde{\tau} = \tilde{\pi} = \tau^* + \frac{\lambda}{1+\lambda}\left[-d' \cdot Y\right]. \tag{13}$$

Where the last term on the right-hand side of Eq. (13) is the additional charge due to the shadow cost of public funds. Note from Eqs. (9) and (13) that an increase in the slope of the total demand has two opposite effects: On the one hand, it reduces τ^* because it raises the inspection cost savings, but on the other, it increases the charge due to costly public funds. Furthermore, when the total demand is perfectly inelastic, we get $\tilde{\tau} = \tilde{\pi} = 0$ if the former effect dominates and $\tilde{\tau} = \tilde{\pi} = 1$ (i.e., the maximum EVF) if the cost of public funds does it. This illustrates that, under certain conditions, fare evasion can create a strong incentive to reduce public transport prices that may overcome the cost of public funds. However, when the cost of public funds dominates, the optimal fare is set above the standard marginal-cost pricing rule to reduce the need for costly taxes.

4.3. Optimal frequency

From the first-order conditions, we get the optimal frequency rule given by Eq. (14), where $\tilde{Y}^i$ denotes the optimal number of formal or informal users:

$$OC_f = -\tilde{Y}^P \cdot GC_f + \Pi^e_f. \tag{14}$$

The social planner increases the frequency until the rise in the operating costs equals the reduction in the formal users' cost plus the change in the profit of the inspection agency, which is given by:

$$\Pi^e_f = -[\tilde{Y}^e - IC_\pi][GC_f + EC_f]. \tag{15}$$

Where the term in the first square brackets of Eq. (15) is the difference between the marginal revenues from fines $\tilde{Y}^e$ (recalling that the fine equals one), and the marginal inspection cost IC_π; while the term in the second square brackets is the change in the costs perceived by evaders due to a marginal increase in the frequency.

Note that optimal frequency is obtained holding $\tilde{Y}^i$ constant. This is correct because the optimal pricing and inspection rules, given by Eq. (12), ensure that any marginal benefits or costs of increasing the frequency via changes in the number of users have zero effect on social welfare (Small and Verhoef, 2007, p. 164). In other words, the social planner captures any variation in the operating and users' costs due to a demand change through the optimal fare and inspection rate. This has two consequences: first, the shadow cost of public funds affects the optimal frequency rule via changes in the optimal price and inspection rate (Hörcher et al., 2020; Zhang et al., 2020; Sun et al., 2016); second, the effect of fare evasion on the optimal frequency is captured by Π^e_f.

[9] The demand *superelasticity*, first derived by Boiteux (1956), is usually defined in terms of the direct and cross demand elasticities as:

$$\xi_i = \frac{\eta_{ii}\eta_{jj} - \eta_{ij}\eta_{ji}}{\eta_{ji} - \eta_{jj}} > 0,$$

where $\eta_{ii} = \frac{\partial Y^i}{\partial p^i}\frac{p^i}{Y^i}$ and $\eta_{ij} = \frac{\partial Y^i}{\partial p^j}\frac{p^j}{Y^i}$ (Laffont and Tirole, 1993). In Appendix B, we show that the demand *superelasticity* can be expressed in terms of the benefit function as

$$\xi_i = \frac{p^i}{-B_{ii} \cdot Y^i - B_{ji} \cdot Y^j} > 0, \quad i \neq j.$$

To illustrate the latter, let us assume for a moment that the cost perceived by evaders does not change with quality (i.e., $GC_f + EC_f = 0$); therefore, increasing the frequency does not change the inspection profit (i.e., $\Pi^e_f = 0$). In this case, Eq. (14) becomes the standard frequency rule $OC_f = -Y^p \cdot GC_f$. Hence, Π^e_f captures the distortion in the optimal frequency rule that emerges as a consequence of fare evasion. If it is positive, fare evasion sums an additional benefit to the usual public transport frequency rule that induces higher service quality; if it is negative, fare evasion has the opposite effect.

The fare evasion literature suggests that improving the frequency increases the evader's cost and, therefore, reduces fare evasion (Busco et al., 2021; Barabino et al., 2015; Guzman et al., 2021; Guarda et al., 2016a,b). In this case, Π^e_f is positive if reducing the inspection rate increases the inspection profit. Thus, we can summarize the impact of fare evasion in the optimal frequency as follows: *If improving the frequency of public transport reduces informal demand, fare evasion increases the optimal public transport frequency as long as the marginal inspection cost exceeds the marginal revenue of fines.*

The intuition behind this result is the following: if increasing the frequency makes evasion less attractive, the social planner captures this effect through a reduction in the inspection rate, which in turn, increases social welfare if the fall in the inspection cost IC_π is larger than the fall in revenues from fines, $\tilde{Y}^e$. In other words, if fare evaders are worse off when the frequency increases, the social planner uses service quality to control fare evasion and save money on inspection. This suggests that transport authorities may have strong incentives to improve service quality when the inspection is too costly or ineffective or when revenues from fines are negligible.[10]

4.4. Revisiting the Mohring effect and public transport subsidies

An important result in the public transport literature is the so-called Mohring effect (1972, 1976). It states that, under certain technical conditions, the sum of operating and users' costs grows less than proportional with demand, leading to an optimal fare that does not cover operating costs and requires subsidies (Silva, 2021). The source of the Mohring effect is that the optimal frequency increases with demand; however, since previous studies do not consider fare evasion, it is relevant to analyze if this result holds in our model. To do this, let us treat quantities Y^p and Y^e as parameters. Then, totally differentiating the optimal frequency rule, we get:

$$\frac{df}{dY^p} = -\frac{W_{fp}}{W_{ff}} = -\frac{\Pi^e_{fp} - \left[GC_f + Y^p \cdot GC_{fY} + OC_{fY}\right]}{\Pi^e_{ff} - \left[Y^p \cdot GC_{ff} + OC_{ff}\right]}. \tag{16}$$

The denominator on the right-hand side of Eq. (16) is negative to satisfy the second-order conditions. Therefore, the sign of df/dY^p is given by the sign of the welfare's cross-derivative W_{fp}, which is positive if the following condition holds:

$$\Pi^e_{fp} > GC_f + Y^p GC_{fY} + OC_{fY}. \tag{17}$$

Thus, whether the Mohring effect is active or not depends on the functional forms of the cost functions, specifically, on the second derivatives of generalized, evasion, inspection, and operating costs.[11]

Now, let us replace the optimal price in the financial result of the public transport operator to get the optimal subsidy per formal user:

$$S = \underbrace{OC/Y^p - [OC_Y + Y^p \cdot GC_Y]}_{\text{Mohring Effect (+)}} + \underbrace{\{-IC_p - Y^e[GC_Y + EC_Y]\}}_{\text{Fare evasion (+)}} + \underbrace{\frac{\lambda}{1+\lambda}\left[B_{pp} \cdot Y^p + B_{ep} \cdot Y^e\right]}_{\text{Cost of public funds (-)}}. \tag{18}$$

The first component of the optimal subsidy is the difference between the average and marginal costs. It is the optimal subsidy when fare evasion and costly public funds are not considered in the analysis (Jara-Díaz and Gschwender, 2005); thus, it is positive if the Mohring effect is active. The second component is the marginal benefit of reducing the fare due to evasion, which is positive and has two terms: the inspection cost savings and the fall in the evaders' total costs. Finally, the third component is negative since it is the cost of subsidizing public transport in terms of distortionary taxes. When $\lambda = 0$, this distortion equals zero because subsidies are just transfers between the government and the operator; however, if $\lambda > 0$, it reduces the optimal subsidy.

Then, we have two forces that put upward pressure on public transport subsidization: the Mohring effect and the benefits of reducing fare evasion. On the other hand, the cost of public funds pushes in the opposite direction. Therefore, the subsidy amount, if needed, will depend on which of these effects dominates.

[10] If we assume that π is the disutility of an anti-evasion activity that does not generate revenues (e.g., awareness campaigns or deterrence measures), the marginal profit is given by $\Pi^e_f = IC_\pi[GC_f + EC_f] > 0$. Therefore, in this case, the social planner has incentives to improve service quality as long as it reduces fare evasion.

[11] In a world without evasion, the optimal frequency increases with demand if the right-hand side of condition (17) is negative (Hörcher and Tirachini, 2021; Gómez-Lobo, 2014). Of course, this depends on the functional forms of the operating and generalized costs. For example, the optimal frequency follows the well-known square root formula when the generalized cost is linear in the number of users and the headway, and the operating cost is linear in the frequency (i.e., $GC_{fY} < 0$, $GC_f < 0$ and $OC_{fY} = 0$).

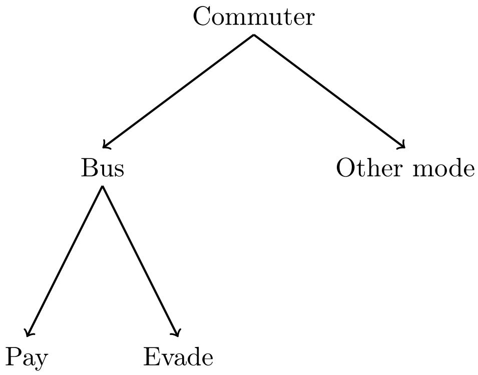

Fig. 1. Commuters' Choice Tree.

5. Numerical analysis

In the previous section, we found analytical expressions that characterize public transport's optimal price, inspection rate, and frequency. However, since we do not define functional forms for the demand and cost functions, we cannot find closed-form formulas for the decision variables to quantify how fare evasion impacts the optimal public transport provision. Thus, in this section, we perform numerical simulations aiming to reflect a representative bus system with a high evasion rate. To do this, we introduce two changes to our theoretical model. First, we represent the individual's choice using a Nested-Logit (NL), a Random Utility Model (RUM) typically used in the transport literature (Ortúzar and Willumsen, 2011; Train, 2009). Second, we consider endogenous bus capacity K and a bus occupancy restriction. Appendices C and D prove that our main analytical results hold with these changes.

The NL has a simple functional form that allows different substitution patterns by grouping correlated or similar alternatives into nests. It also has an analytical expression for the consumers' surplus, making it useful in the welfare analysis of public transport (see, e.g., Hörcher and Graham, 2020; Basso and Silva, 2014). For simplicity, we assume a representative commuter with the decision process presented in Fig. 1: first, the commuter chooses whether to use the bus or not, and then, those who choose the bus decide between paying or evading the fare.

In line with our theoretical model, the utility functions of paying and evading are given by:

$$V^p = \theta^p - \tau - GC, \tag{19}$$

$$V^e = \theta^e - \pi \cdot h - GC - EC, \tag{20}$$

where the alternative specific constants θ^p and θ^e represent the valuation of traveling formally and informally, and h is the value of the fine. Then, normalizing the utility of not using the bus to zero, the probability that the representative commuter chooses alternative $i \in \{p, e\}$ is given by:

$$\Phi^i = \frac{\exp\left(\frac{V^i}{\eta}\right)}{\sum_{j\in\{p,e\}} \exp\left(\frac{V^j}{\eta}\right)} \cdot \frac{\exp \eta \cdot \ln\left(\sum_{j\in\{p,e\}} \exp\left(\frac{V^j}{\eta}\right)\right)}{\exp \eta \cdot \ln\left(\sum_{j\in\{p,e\}} \exp\left(\frac{V^j}{\eta}\right)\right) + 1}, \tag{21}$$

where the parameter $\eta \in (0, 1]$ is a measure of the degree of substitution between formal and informal demand (i.e., the smaller η is, the greater the degree of substitution).

Assuming that the users' expenditure on transport is sufficiently small for the income effect to be negligible, the consumers' surplus can be computed using the well-known log sum formula (Small and Rosen, 1981; Anderson and de Palma, 1992; Batley and Dekker, 2019):

$$CS = N \cdot \ln\left(\exp \eta \cdot \ln \sum_{j\in\{p,e\}} \exp\left(\frac{V^j}{\eta}\right) + 1\right). \tag{22}$$

Following an extension of Jansson's model (1980, 1984) proposed by Jara-Díaz and Gschwender (2003), we consider a bus line of L kilometers and a cycle time equal to:

$$t_c = T + t\frac{Y}{f}, \tag{23}$$

where T is the in-vehicle time, and t is the average boarding and alighting time per user. The commuters travel a distance l. Thus, the generalized cost is given by:

$$GC = \alpha_w \frac{\delta}{f} + \alpha_v \frac{l}{L}\left[T + t\frac{Y}{f}\right]. \tag{24}$$

The first term is the waiting time cost, computed as a fraction of the headway multiplied by the value of waiting time savings α_w. The second term is the in-vehicle time cost, and it is computed as the time that users travel on-board the bus multiplied by the value of in-vehicle time savings α_v, which is a linear function of the bus occupancy ϕ:

$$\alpha_v = \alpha_0 + \alpha_1 \phi, \tag{25}$$

where:

$$\phi = \frac{Yl}{fKL} \leq 1, \tag{26}$$

Finally, Jara-Díaz and Gschwender (2003) compute the operating costs as the product of the fleet size and the cost per bus:

$$OC = \gamma \cdot [c_0 + c_1 K][fT + tY], \tag{27}$$

where γ is an operating cost multiplier used for sensitivity analysis.

We assume a personal evasion cost with constant elasticity with respect to boardings and bus occupancy:

$$EC = \beta_0 \cdot b^{\beta_1} \cdot \phi^{\beta_2}, \tag{28}$$

where β_0, β_1, and β_2 are parameters calibrated to represent the commuters' evading behavior and b is the number of users boarding at each bus stop, which are separated by a distance s:

$$b = \frac{Ys}{fL}. \tag{29}$$

Finally, the inspection cost is given by a linear function of the number of inspections per hour:

$$IC = \omega \cdot [\pi \cdot Y], \tag{30}$$

where ω is the marginal inspection cost and $\pi \cdot Y$ is the number of inspections per hour.

The parameters used in the simulations are presented in Table 1. The cycle time, generalized cost, and inspection parameters are adapted from previous literature (Fielbaum et al., 2020; Jara-Díaz and Gschwender, 2003; Manríquez, 2019; Jara-Díaz and Gschwender, 2009). The remaining parameters are calibrated to comply with the baseline scenario described in Table 2, which aims to reflect a representative bus system with a high evasion rate.[12]

Fig. 2 summarizes the results of the numerical simulations. The upper panel illustrates the optimal frequency and the cost recovery ratio (i.e., the percentage of the operating costs covered by the fare), while the lower panel shows the resulting market share and evasion rate of the bus system. The baseline scenario is marked by point B and the optimal solution for the parameters presented in Table 1 is marked by Point O. The color lines show the sensitivity with respect to: (i) the operating cost multiplier γ, (ii) the Shadow Cost of Public Funds (SCPF) λ, (iii) the marginal inspection cost ω, and (iv) the evasion cost elasticity with respect to boardings β_1.

First, let us compare the baseline scenario with the optimal solution. Note that the market share in the optimal solution is only two percentage points lower than in the baseline scenario; however, the evasion rate falls from 30% to less than 20%. This means that the total number of users in the baseline scenario is close to optimal; however, the proportion of these trips that do not pay the fare is ten percentage points above the welfare-maximizing level. Thus, the optimal inspection rate is five times higher than in the baseline scenario. The main consequence of this reduction in the evasion rate is that revenues from fares increase by 20%, so the cost recovery ratio rises from 60% in the baseline scenario to 75% in the optimal solution. This means that the public transport subsidization in the baseline scenario is above the optimal level due to the revenue losses from fare evasion. Finally, note that reducing fare evasion allows the social planner to increase the frequency by 15%, improving the service quality perceived by all users.

Unlike previous literature that considers the cost of public funds (see, e.g., Börjesson et al., 2019; Basso and Silva, 2014; Proost and van Dender, 2008; Hörcher et al., 2020), the optimal fare does not cover the operating cost in all our simulations. This occurs for two reasons: first, straightforward computations show that the Mohring effect is active, and second, increasing subsidies reduces inspection and evaders' costs. As shown in Eq. (18), these two forces create an incentive to raise the operational subsidy that, in our simulations, dominates the cost of public funds. On the other hand, the marginal inspection profit decreases with frequency. As explained in Eq. (14), this creates an incentive to reduce the frequency; however, the effect of this incentive is relatively small and does not significantly reduce the optimal service quality.

As expected, the optimal frequency and market share decrease with the operating cost multiplier $\gamma \in [0.5, 1.5]$. However, the effect of γ on the cost recovery ratio is less significant because both the fare and the operating cost grow proportionally with γ due to the linear nature of the operating costs function. Nevertheless, there is an indirect effect due to the Mohring effect and the incentive to reduce fare evasion that explains the 13 percentage points fall in the cost recovery ratio observed in the sensitivity analysis. Interestingly, the optimal evasion rate remains almost unchanged in the range of γ. This happens because, as shown in

[12] The parameters presented in Table 1 are a unique solution to the calibration problem for a reasonable domain of the variables. Most formal and informal demand values and elasticities are based on studies conducted in Santiago (Batarce and Galilea, 2018; de Grange et al., 2013; Guarda et al., 2016b,a). However, for calibration purposes, the informal demand elasticities with respect to price and inspection rate are higher than the values reported for this city.

Table 1
Simulation parameters.

Parameter	Value	Unit	Description
Nested Logit parameters			
N	12000	Pax/h	Number of potential bus users (commuters)
θ^p	0.34	\$	Valuation of making the trip formally
θ^e	0.03	\$	Valuation of making the trip informally
η	0.87		Nest parameter
Cycle time parameters			
T	2.72	h	Time in motion in each cycle
t	5	s	Average boarding and alighting time per user
L	60	km	Cycle length
s	0.5	km	Distance between bus stops
Generalized cost parameters			
α_w	2.96	\$/h	Value of waiting time savings
α_0	1.18	\$/h	Base value of in-vehicle time savings
α_1	0.30	\$/h	Bus occupancy parameter
δ	0.5		Fraction of headway
l	10	km	Travel distance
Personal evasion cost parameters			
β_0	10.59		Scale factor
β_1	−1.65		Elasticity w.r.t. boardings
β_2	−0.17		Elasticity w.r.t. occupancy
Operating costs parameters			
γ	1		Operating cost multiplier
c_0	24.56	\$/h	Base cost per vehicle hour
c_1	0.49	\$/h	Vehicle size cost
Inspection parameters			
ω	1.5	\$/inspection	Marginal inspection cost
h	50	\$	Value of the fine
Welfare parameters			
λ	0.15		Shadow cost of public funds

Table 2
baseline scenario.

Variable		Value	Unit
Baseline Demand			
Market share of the bus	Y/N	0.30	
Evasion rate	Y^e/Y	0.30	
Formal demand elasticity w.r.t. price	$\%\Delta Y^p/\%\Delta\tau$	−0.35	
Informal demand elasticity w.r.t. boardings	$\%\Delta Y^e/\%\Delta b$	1.096	
Informal demand semi-elasticity w.r.t. Occupancy	$\%\Delta Y^e/\Delta\phi$	0.008	
Baseline supply			
Cost recovery ratio	$\tau \cdot Y^p/OC$	0.60	
Frequency	f	6	bus/h
Bus size	K	100	pax
Bus occupancy	ϕ	1	
Fare	τ	0.37	\$
Operating cost ratio	c_0/c_1	50	
Inspection rate	π	0.001	

Eq. (11), the optimal inspection rate rises as the optimal fare increases. Therefore, an increase in the operating costs reduces formal and informal demand similarly, leading to an evasion rate that remains almost constant when γ increases.

In line with the previous literature, the shadow cost of public funds strongly affects the optimal fare, ranging from \$0.24 when $\lambda = 0$ to \$0.53 when $\lambda = 0.3$, which in turn, strongly impacts the optimal market share and frequency (Sun et al., 2016; Sun and Szeto, 2019; de Palma et al., 2017; Zou and Mizokami, 2014). However, unlike in previous studies, the optimal fare does not cover the operating cost for realistic values of the shadow cost of public funds, which range between 0.08 and 0.20 depending on the country and the tax considered (Harrison et al., 2002; Ballard et al., 1985; Auriol and Warlters, 2012). Note that, again, the evasion rate remains almost the same. This occurs because the optimal fare and inspection rate are proportional to $\lambda/[1+\lambda]$, as shown in Eq. (12). Therefore, an increase in the cost of public funds reduces formal and informal demand similarly, leading to an optimal evasion rate that does not change significantly with λ.

As expected, the evasion rate increases with the marginal inspection cost $\omega \in [0, 4]$. However, it has a low effect on the optimal frequency and fare. We obtain a 17% evasion rate when ω is zero. In this extreme case, reducing fare evasion through inspection is free, so we might expect that the optimal solution would be to inspect as many users as possible to reduce fare evasion to a

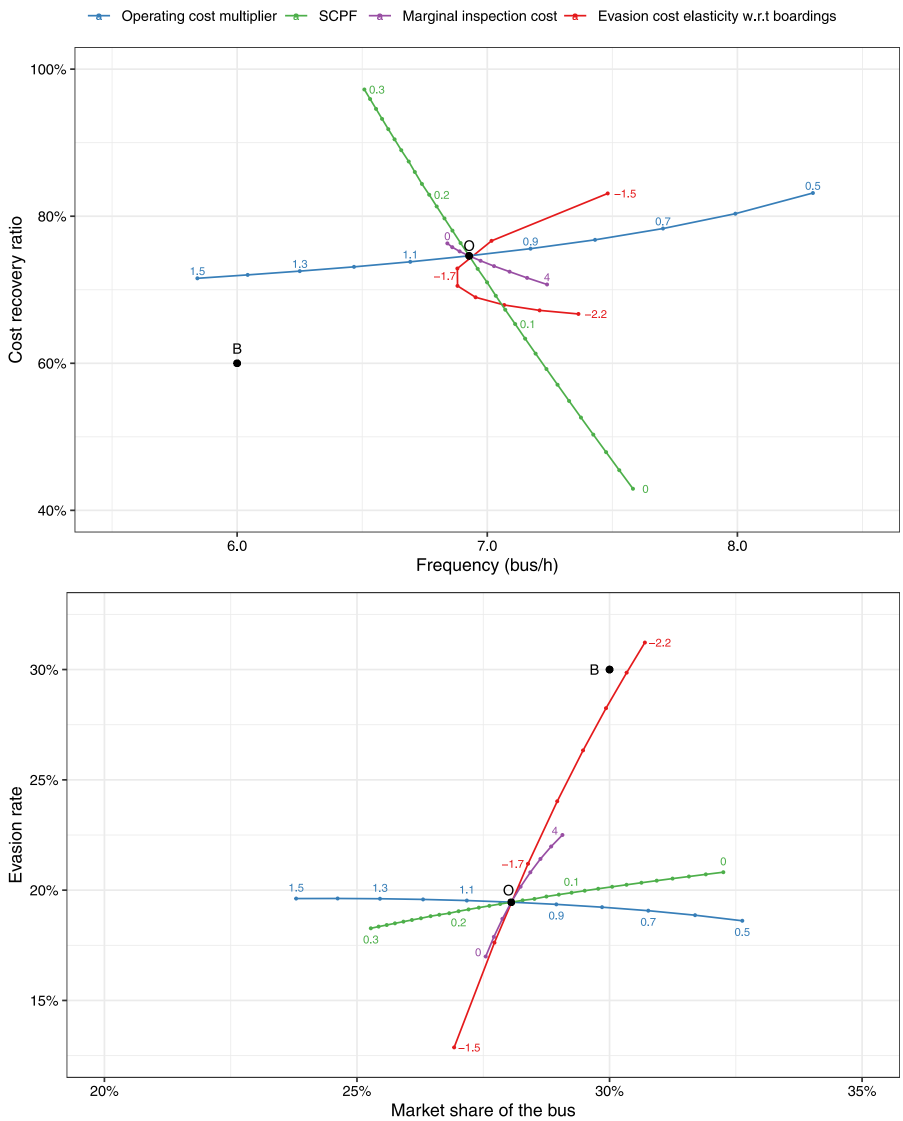

Fig. 2. Numerical results.

minimum. However, the inspection rate only rises from 0.2% to 0.8% in the range of ω. This occurs because some individuals benefit more from evading than paying due to the NL formulation. Therefore, forcing these users to pay or leave the market would produce welfare losses even if the inspection is free.

Finally, let us analyze the impact of the evasion cost elasticity with respect to boardings β_1. First, note that the evasion rate falls by 18 percentage points in the range of $\beta_1 \in [-2.2, -1.5]$. To explain this result, note that the optimal frequency and market share stay close to 7 bus/h and 29%, respectively, meaning that approximately four passengers board the bus at each stop in the range of β_1. However, the more elastic the evasion cost is, the higher the perception of anonymity when boarding with the other three passengers. Therefore, the personal evasion cost decreases with the elasticity, falling from \$1.56 when $\beta_1 = -1.5$ to \$0.45 when $\beta_1 = -2.2$.

Since evasion becomes less costly as the elasticity increases, the optimal fare must be reduced to control evasion. Thus, the cost recovery ratio falls by 17 percentage points in the range of β_1. On the other hand, the optimal frequency only changes by 9%. Although this change is not significant, the optimal frequency follows a U-shape that deserves some discussion. The driving force behind this shape is the change in the marginal inspection profits. Frequency reaches its minimum value when $\beta_1 = -1.7$. At this point, we get the lower marginal inspection profit and, therefore, the maximum incentive to reduce optimal frequency. When $\beta_1 = -2.2$, the marginal effect of increasing the frequency on the evaders is small, so reducing the frequency has a negligible impact on the inspection revenues despite the high evasion rate. On the other hand, when $\beta_1 = -1.5$, the marginal effect of frequency on evaders' costs triples, but the number of evaders falls by one-third. Therefore, the incentive to reduce the frequency is roughly the same for the two extreme values of β_1, leading to similar optimal frequencies.

6. Conclusion

In this paper, we have developed what we believe is the first theoretical model to study how fare evasion affects welfare-maximizing public transport price and quality. The main finding of our analysis is that reducing fare evasion may be a solid new argument for public transport subsidization. Numerical simulations confirm that the incentive to reduce fare evasion and the Mohring effect can justify subsidies considering realistic values of the shadow cost of public funds. The policy implication of this result is that a significant reduction in the public transport price may be an efficient strategy to deal with fare evasion in cities where other measures, such as inspection, awareness campaigns, and deterrence, have proven ineffective or too costly. Moreover, our results contribute to the ongoing policy discussion about public transport subsidization in the developing world, where fare evasion is more relevant for transport authorities and scholars.

Interestingly, a positive evasion rate is optimal even if the marginal inspection cost is zero. This results from including the evaders' surplus in the social welfare function. However, as pointed out by anonymous reviewers, whether this is correct is an ethical and philosophical question without an objective answer. One may argue that the optimal level of fare dodging should be zero due to its unethical nature, so the fact that the utilitarian approach leads to a positive number of evaders may suggest that our so-called social welfare function does not reflect that fare evasion is considered an antisocial activity. Intuition suggests that a social welfare function that captures policymakers' concern for morality and fairness may lead to sterner inspection to reduce fare evasion below what the utilitarian approach implies (Sandmo, 1981). Nevertheless, this issue remains an open question not addressed by the economic literature, and therefore, we believe it is an interesting new avenue for future research.

In our model, evaders heterogeneity is captured in the benefit function, which allows us to obtain simple analytical expressions for the optimal design and pricing. We acknowledge, though, that evaders' heterogeneity is more complex than this simple representation. Therefore, a natural extension to our work is to consider evaders' heterogeneity regarding intentionality, income, and other socio-economic characteristics relevant to the analysis. Spatial variations of the demand conditions may also affect our results since the average crowding experienced by users is higher than the average bus occupancy. This occurs because travelers are disproportionally concentrated in more crowded vehicles, so fare evasion would also be concentrated where crowding is higher. Thus, including a spatial dimension in the analysis could be an interesting extension of our work.

Another possible extension is to consider the automobile as a substitute mode since unpriced car externalities significantly impact public transport design and pricing. Furthermore, a more complex but realistic approach, aiming to represent a specific city, should be considered to study the interaction between fare evasion and other modes, markets, and policies. We also see the study of non-linear pricing, such as travel passes, as a possible extension since the marginal price paid by pass owners is zero.

Our results also rely on the assumption that a public operator provides the bus service. However, public transport is usually operated by private firms whose incomes may be affected by fare evasion. Therefore, regulating private operators can also be an interesting extension of our model. Finally, we acknowledge that fare evasion depends on social, political, and cultural factors beyond the scope of this paper. Although challenging, incorporating these dimensions into fare evasion modeling may be an interesting avenue for future research.

7. Notational glossary

Notation	Description
Decision variables	
τ	Public transport fare
π	Inspection rate
f	Frequency
Y^e	Number of fare evaders
Y^p	Number of formal users
Y	Total number of users
Social Welfare function	
W	Social Welfare
$B(Y^p, Y^e)$	Consumers' benefit function
CS	Consumers' Surplus
λ	Shadow cost of public funds
Π^p	Financial result of the bus operator
Π^e	Financial result of the inspection agency

Cost functions	
$GC(Y, f)$	Generalized cost
$EC(Y, f)$	Personal evasion cost
$OC(Y, f)$	Operating costs
$IC(\pi)$	Inspection costs
Demand elasticities	
η_{ii}	Own price demand elasticity
η_{ij}	Cross price demand elasticity
ξ_i	Demand *superelasticity*

CRediT authorship contribution statement

Raúl Ramos: Conceptualization, Methodology, Formal analysis, Investigation, Writing – original draft,Visualization. **Hugo E. Silva:** Conceptualization, Methodology, Validation, Writing – original draft,Writing – review & editing, Supervision, Project administration, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

We gratefully acknowledge financial support from ANID PIA/PUENTE AFB220003, ANID FONDAP 1522A0002, ANID FONDECYT grant 1200801, ANID BECAS/DOCTORADO NACIONAL 21180398 and Erasmus+ programme of the European Union.

Appendix A. Endogenous value of the fine

Let $h \in [0, \bar{h}]$ denote the endogenous fine, where $\bar{h}$ is the maximum value given by law. Then, the equilibrium condition for the evaders is given by:

$$B_e = \pi h + GC(Y, f) + EC(Y, f), \tag{31}$$

where πh is the Expected Value of the Fine (EVF). Substituting Eq. (31) in the social welfare function Eq. (5), we can write the planner's welfare maximization problem as:

$$\underset{Y^i, f, h}{\text{Max}} \; B(Y^p, Y^e) + \lambda \left[\sum_{i \in \{p,e\}} Y^i \cdot B_i \right] - [1+\lambda] \left[Y \cdot GC(Y, f) + Y^e \cdot EC(Y, f) + OC(Y, f) + IC\left(\frac{B_e - GC(Y, f) - EC(Y, f)}{h} \right) \right], \tag{32}$$

$$\text{s.t.} \quad h \leq \bar{h}. \tag{32a}$$

Note that the objective function in Eq. (32) is strictly increasing with the value of the fine h. Therefore, the fine's restriction Eq. (32a) is always binding.

Then, the optimal fare and EVF, denoted by ($\sim$), are given by the following Ramsey formula for interdependent demand:

$$\xi_p \left[\frac{\tilde{\tau} - \tau^*}{\tilde{\tau}} \right] = \xi_e \left[\frac{\tilde{\pi}\bar{h} - \pi^*\bar{h}}{\tilde{\pi}\bar{h}} \right] = \frac{\lambda}{1+\lambda}, \tag{33}$$

Where τ^* and $\pi^*\bar{h}$ are given by:

$$\tau^* = [OC_Y + Y^p \cdot GC_Y] + Y^e \left[GC_Y + EC_Y \right] + IC_\pi \left[\frac{B_{ep} - GC_Y - EC_Y}{\bar{h}} \right], \tag{34}$$

$$\pi^*\bar{h} = \tau^* - \frac{IC_\pi}{\bar{h}} \left[1 - \frac{B_{ep}}{B_{ee}} \right] [-B_{ee}], \tag{35}$$

while ξ_i denotes the demand *superelasticity*, which is usually defined in terms of the direct demand elasticity η_{ii} and cross demand elasticity η_{ij} as (see Appendix B):

$$\xi_i = \frac{\eta_{ii}\eta_{jj} - \eta_{ij}\eta_{ji}}{\eta_{ji} - \eta_{jj}} > 0. \tag{36}$$

On the other hand, the optimal frequency rule is given by:

$$OC_f = -\tilde{Y}^P GC_f + \Pi_f^e, \tag{37}$$

where:

$$\Pi_f^e = -[\tilde{Y}^e \bar{h} - IC_\pi] \left[\frac{GC_f + EC_f}{\bar{h}} \right]. \tag{38}$$

Finally, note that assuming $\bar{h} = 1$ does not affect the optimal design and pricing rules since we are just scaling all the prices by the factor $1/\bar{h}$.

Appendix B. Demand superelasticity

Let us consider a benefit function for two goods $B(Y^i, Y^j)$. In equilibrium, the inverse demand function of each good must be equal to its price:

$$B_i = p^i, \quad \forall i \in \{i, j\}. \tag{39}$$

Differentiating Eqs. (39) with respect to p^i we get:

$$\begin{aligned} B_{ii}\frac{\partial Y^i}{\partial p^i} + B_{ij}\frac{\partial Y^j}{\partial p^i} \quad &= 1, \\ B_{ji}\frac{\partial Y^i}{\partial p^i} + B_{jj}\frac{\partial Y^j}{\partial p^i} \quad &= 0. \end{aligned} \tag{40}$$

Solving system (40) we get:

$$Y_i^i = \frac{\partial Y^i}{\partial p^i} = \frac{B_{jj}}{B_{ii}B_{jj} - B_{ij}B_{ji}}, \tag{41}$$

$$Y_i^j = \frac{\partial Y^j}{\partial p^i} = \frac{-B_{ji}}{B_{ii}B_{jj} - B_{ij}B_{ji}}. \tag{42}$$

On the other hand, the demand *superelasticity* is given by (Laffont and Tirole, 1993):

$$\xi_i = \frac{\eta_{ii}\eta_{jj} - \eta_{ij}\eta_{ji}}{\eta_{ij} - \eta_{jj}} = \frac{p^i\left[Y_i^i Y_j^j - Y_j^i Y_i^j\right]}{Y^j Y_j^i - Y^i Y_j^j}. \tag{43}$$

Replacing Eqs. (41) and (42) in Eq. (43), we get:

$$\xi_i = -\frac{p^i}{Y^i B_{ii} + Y^j B_{ji}}. \tag{44}$$

Appendix C. Analytical solution for the NL

The planner's welfare maximization problem is given by:

$$\underset{Y^i,\tau,\pi,f}{\text{Max}} \qquad CS + [1+\lambda]\cdot[\Pi^p + \Pi^e] \tag{45}$$

$$\text{subject to} \;\; Y^i = \Phi^i, \quad \forall i \in \{p, e\}. \tag{45a}$$

Where, without loss of generality, we normalize the number of commuters and the fine to one. Then, the first-order conditions can be written as:

$$\xi_p\left[\frac{\tilde{\tau} - \tau^*}{\tilde{\tau}}\right] = \xi_e\left[\frac{\tilde{\pi} - \pi^*}{\tilde{\pi}}\right] = \frac{\lambda}{1+\lambda}, \tag{46}$$

$$OC_f = -\tilde{Y}^P GC_f + \Pi_f^e. \tag{47}$$

Where:

$$\tau^* = [OC_Y + Y^p \cdot GC_Y] + Y^e\left[GC_Y + EC_Y\right] - IC_\pi\left[GC_Y + EC_Y + \frac{\Phi_\tau^e}{\Phi_\tau^p\Phi_\pi^e - \Phi_\pi^p\Phi_\tau^e}\right], \tag{48}$$

$$\Phi_\tau^i = \frac{\partial \Phi^i}{\partial \tau}, \quad \Phi_\pi^i = \frac{\partial \Phi^i}{\partial \pi}, \tag{49}$$

$$\pi^* = \tau^* - IC_\pi \cdot \frac{\eta}{\Phi^e}, \tag{50}$$

$$\frac{\tau}{\xi_p} = \frac{\pi}{\xi_e} = \frac{1}{1 - [\Phi^p + \Phi^e]}. \tag{51}$$

As expected, the optimal price and design rules (i.e., Eqs. (46) and (47)) are equivalent to the ones obtained in Section 4.

Appendix D. Endogenous bus capacity

Let us define $GC(Y, f, \phi(Y, f, K))$, $EC(Y, f, \phi(Y, f, K))$ and $OC(Y, f, K)$ as the generalized, evasion and operating cost functions, and $\phi(Y, f, K)$ as the occupancy rate. Then, the planner's welfare maximization problem is given by:

$$\max_{Y^i, f, K} \quad B(\cdot) + \lambda \left[\sum_{i \in \{p,e\}} Y^i \cdot B_i \right] - [1+\lambda] \Big[Y \cdot GC(\cdot) + Y^e \cdot EC(\cdot) + OC(\cdot) + IC(\cdot) \Big] \tag{52}$$

$$\text{subject to} \quad \phi(\cdot) \le \phi_m. \tag{52a}$$

From the Karush–Kuhn–Tucker conditions of the planner's maximization problem, it is straightforward to prove that restriction (52a) is active if:

$$OC_K \Big|_{\bar{K}(\cdot)} \ge -Y^p \cdot GC_K \Big|_{\phi_m,} + \Pi^e_K \Big|_{\phi_m}, \tag{53}$$

where:

$$\bar{K}(Y, f) = \phi^{-1}(Y, f, \phi_m). \tag{54}$$

Otherwise, the occupancy restriction is not binding.

D.1. Not binding occupancy restriction

When the occupancy restriction is not binding, the optimal fare and frequency are given by Eqs. (12) and (14), while the optimal bus size is given by:

$$OC_K = -Y^p \cdot GC_K \cdot + \Pi^e_K, \tag{55}$$

which is analogous to the optimal frequency rule.

D.2. Binding occupancy restriction

Let us define $\overline{GC}$, $\overline{EC}$ and $\overline{OC}$ as the generalized, evasion and operating cost when occupancy restriction is binding:

$$\overline{GC}(Y, f) = GC(Y, f, \phi_m), \tag{56}$$

$$\overline{EC}(Y, f) = EC(Y, f, \phi_m), \tag{57}$$

$$\overline{OC}(Y, f) = OC(Y, f, \bar{K}(Y, f)). \tag{58}$$

Then, the planner's maximization problem with binding occupancy restriction is equivalent to Eq. (8)

References

Adler, M.W., Liberini, F., Russo, A., van Ommeren, J.N., 2021. The congestion relief benefit of public transit: Evidence from Rome. J. Econ. Geogr. 21, 397–431. http://dx.doi.org/10.1093/jeg/lbaa037.

Adler, M.W., van Ommeren, J.N., 2016. Does public transit reduce car travel externalities? Quasi-natural experiments' evidence from transit strikes. J. Urban Econ. 92, 106–119. http://dx.doi.org/10.1016/j.jue.2016.01.001.

Allen, J., Muñoz, J.C., Ortúzar, J.d.D., 2019. On evasion behaviour in public transport: Dissatisfaction or contagion? Transp. Res. A 130, 626–651. http://dx.doi.org/10.1016/j.tra.2019.10.005.

Allport, R., 1981. The costing of bus, light rain transit and metro public transport systems. Traffic Eng. Control 22.

Anderson, S.P., de Palma, A., 1992. Multiproduct firms: A nested logit approach. J. Ind. Econ. 40, 261. http://dx.doi.org/10.2307/2950539.

Anderson, S.P., de Palma, A., Thisse, J.F., 1988. A representative consumer theory of the logit model. Internat. Econom. Rev. 29, 461. http://dx.doi.org/10.2307/2526791.

Anderson, S.P., de Palma, A., Thisse, J.F., 1989. Demand for differentiated products, discrete choice models, and the characteristics approach. Rev. Econom. Stud. 56, 21–35. http://dx.doi.org/10.2307/2297747.

Auriol, E., Warlters, M., 2012. The marginal cost of public funds and tax reform in Africa. J. Dev. Econ. 97, 58–72. http://dx.doi.org/10.1016/j.jdeveco.2011.01.003.

Bae, S.H., Choi, J.P., 2006. A model of piracy. Inf. Econ. Policy 18, 303–320. http://dx.doi.org/10.1016/J.INFOECOPOL.2006.02.002.

Ballard, B.C.L., Shoven, J.B., Whalley, J., 1985. General equilibrium computations of the marginal welfare costs of taxes in the United States. Am. Econ. Rev. 75, 128–138.

Banerjee, D.S., 2003. Software piracy: a strategic analysis and policy instruments. Int. J. Ind. Organ. 21, 97–127. http://dx.doi.org/10.1016/S0167-7187(01)00090-X.

Bansal, P., Hörcher, D., Graham, D.J., 2022. A dynamic choice model to estimate the user cost of crowding with large-scale transit data. J. Roy. Statist. Soc. Ser. A 1–25. http://dx.doi.org/10.1111/rssa.12804.

Bansal, P., Hurtubia, R., Tirachini, A., Daziano, R.A., 2019. Flexible estimates of heterogeneity in crowding valuation in the New York City subway. J. Choice Model. 31, 124–140. http://dx.doi.org/10.1016/j.jocm.2019.04.004.

Barabino, B., Lai, C., Olivo, A., 2020. Fare evasion in public transport systems: a review of the literature. Public Transp. 12, 27–88. http://dx.doi.org/10.1007/s12469-019-00225-w.

Barabino, B., Salis, S., 2019. Moving towards a more accurate level of inspection against fare evasion in proof-of-payment transit systems. Netw. Spat. Econ. 1–28. http://dx.doi.org/10.1007/s11067-019-09468-3.

Barabino, B., Salis, S., 2020. Do students, workers, and unemployed passengers respond differently to the intention to evade fares? An empirical research. Transp. Res. Interdiscip. Perspect. 7, 100215. http://dx.doi.org/10.1016/j.trip.2020.100215.
Barabino, B., Salis, S., Useli, B., 2013. A modified model to curb fare evasion and enforce compliance: Empirical evidence and implications. Transp. Res. A 58, 29–39. http://dx.doi.org/10.1016/j.tra.2013.10.007.
Barabino, B., Salis, S., Useli, B., 2014. Fare evasion in proof-of-payment transit systems: Deriving the optimum inspection level. Transp. Res. B 70, 1–17. http://dx.doi.org/10.1016/j.trb.2014.08.001.
Barabino, B., Salis, S., Useli, B., 2015. What are the determinants in making people free riders in proof-of-payment transit systems? Evidence from Italy. Transp. Res. A 80, 184–196. http://dx.doi.org/10.1016/j.tra.2015.07.017.
Barabino, B., Salis, S., Useli, B., 2022. Assessing the intention to evade fares for demographic segments of passengers: Empirical research in Italy for building smart(er) cities. J. Urban Plann. Dev. 148, http://dx.doi.org/10.1061/(asce)up.1943-5444.0000804.
Basso, L.J., Jara-Díaz, S., 2012. Integrating congestion pricing, transit subsidies and mode choice. Transp. Res. A 46, 890–900. http://dx.doi.org/10.1016/j.tra.2012.02.013.
Basso, L.J., Silva, H.E., 2014. Efficiency and substitutability of transit subsidies and other urban transport policies. Am. Econ. J. Econ. Policy 6, 1–33. http://dx.doi.org/10.1257/pol.6.4.1.
Batarce, M., Galilea, P., 2018. Cost and fare estimation for the bus transit system of Santiago. Transp. Policy 64, 92–101. http://dx.doi.org/10.1016/j.tranpol.2018.01.002.
Batley, R., Dekker, T., 2019. The intuition behind income effects of price changes in discrete choice models, and a simple method for measuring the compensating variation. Environ. Resour. Econ. 74, 337–366. http://dx.doi.org/10.1007/s10640-019-00321-2.
Besfamille, M., Figueroa, N., Guzmán, L., 2022. Fare evasion and monopoly regulation. SSRN Electron. J. http://dx.doi.org/10.2139/ssrn.4043316.
Boiteux, M., 1956. Sur la gestion des Monopoles Publics astreints a l'equilibre budgetaire. Econometrica 24, 22. http://dx.doi.org/10.2307/1905256.
Bonfanti, G., Wagenknecht, T., 2010. Human factors reduce aggression and fare evasion. Public Transp. Int. 59, 28–32.
Börjesson, M., Eliasson, J., Rubensson, I., 2020. Distributional effects of public transport subsidies. J. Transp. Geogr. 84, 102674. http://dx.doi.org/10.1016/j.jtrangeo.2020.102674.
Börjesson, M., Fung, C.M., Proost, S., 2017. Optimal prices and frequencies for buses in Stockholm. Econ. Transp. 9, 20–36. http://dx.doi.org/10.1016/j.ecotra.2016.12.001.
Börjesson, M., Fung, C.M., Proost, S., Yan, Z., 2019. Do small cities need more public transport subsidies than big cities? J. Transp. Econ. Policy 53, 275–298.
Boyd, C., 1989. Fare evasion and non-compliance: a simple model. J. Transp. Econ. Policy 23, 189–197.
Bucciol, A., Landini, F., Piovesan, M., 2013. Unethical behavior in the field: Demographic characteristics and beliefs of the cheater. J. Econ. Behav. Organ. 93, 248–257. http://dx.doi.org/10.1016/j.jebo.2013.03.018.
Buehler, S., Halbheer, D., Lechner, M., 2017. Payment evasion. J. Ind. Econ. 65, 804–832. http://dx.doi.org/10.1111/joie.12144.
Busco, C., González, F., Jaqueih, Y., Jiménez, F., Alonso, B., 2021. Understanding transantiago users' motivations for paying or evading payment of bus fares. J. Public Transp. 23, http://dx.doi.org/10.5038/2375-0901.23.2.2.
Cantillo, A., Raveau, S., Muñoz, J.C., 2022. Fare evasion on public transport: Who, when, where and how? Transp. Res. A 156, 285–295. http://dx.doi.org/10.1016/j.tra.2021.11.027.
Clarke, R.V., Contre, S., Petrossian, G., 2010. Deterrence and fare evasion: Results of a natural experiment. Secur. J. 23, 5–17. http://dx.doi.org/10.1057/sj.2009.15.
Cools, M., Fabbro, Y., Bellemans, T., 2018. Identification of the determinants of fare evasion. Case Stud. Transp. Policy 6, 348–352. http://dx.doi.org/10.1016/j.cstp.2017.10.007.
Currie, G., Delbosc, A., 2017. An empirical model for the psychology of deliberate and unintentional fare evasion. Transp. Policy 54, 21–29. http://dx.doi.org/10.1016/j.tranpol.2016.11.002.
Dai, Z., Galeotti, F., Villeval, M.C., 2018. Cheating in the lab predicts fraud in the field: An experiment in public transportation. Manage. Sci. 64, 1081–1100. http://dx.doi.org/10.1287/mnsc.2016.2616.
de Grange, L., González, F., Muñoz, J.C., Troncoso, R., 2013. Aggregate estimation of the price elasticity of demand for public transport in integrated fare systems: The case of Transantiago. Transp. Policy 29, 178–185. http://dx.doi.org/10.1016/j.tranpol.2013.06.002.
de Palma, A., Lindsey, R., Monchambert, G., 2017. The economics of crowding in rail transit. J. Urban Econ. 101, 106–122. http://dx.doi.org/10.1016/j.jue.2017.06.003.
Delbosc, A., Currie, G., 2016a. Cluster analysis of fare evasion behaviours in Melbourne, Australia. Transp. Policy 50, 29–36. http://dx.doi.org/10.1016/j.tranpol.2016.05.015.
Delbosc, A., Currie, G., 2016b. Four types of fare evasion: A qualitative study from Melbourne, Australia. Transp. Res. F 43, 254–264. http://dx.doi.org/10.1016/j.trf.2016.09.022.
Delbosc, A., Currie, G., 2019. Why do people fare evade? A global shift in fare evasion research. Transp. Rev. 39, 376–391. http://dx.doi.org/10.1080/01441647.2018.1482382.
Egu, O., Bonnel, P., 2020. Can we estimate accurately fare evasion without a survey? Results from a data comparison approach in Lyon using fare collection data, fare inspection data and counting data. Public Transp. 12, 1–26. http://dx.doi.org/10.1007/s12469-019-00224-x.
Fielbaum, A., Jara-Díaz, S., Gschwender, A., 2020. Beyond the Mohring effect: Scale economies induced by transit lines structures design. Econ. Transp. 22, 100163. http://dx.doi.org/10.1016/j.ecotra.2020.100163.
Fielbaum, A., Jara-Diaz, S., Gschwender, A., 2021. Lines spacing and scale economies in the strategic design of transit systems in a parametric city. Res. Transp. Econ. 90, 100991. http://dx.doi.org/10.1016/j.retrec.2020.100991.
Goerke, L., 2017. Tax evasion in a Cournot oligopoly with endogenous entry. Int. Tax Public Financ. 24, 754–779. http://dx.doi.org/10.1007/s10797-016-9434-z.
Gómez-Lobo, A., 2014. Monopoly, subsidies and the Mohring effect: A synthesis. Transp. Rev. 34, 297–315. http://dx.doi.org/10.1080/01441647.2014.902871.
Gómez-Lobo, A., Tirachini, A., Gutierrez, I., 2022. Optimal prices for ridesourcing in the presence of taxi, public transport and car competition. Transp. Res. C 137, 103591. http://dx.doi.org/10.1016/j.trc.2022.103591.
González, F., Busco, C., Codocedo, K., 2019. Fare evasion in public transport: Grouping Transantiago users' behavior. Sustainability (Switzerland) 11, 6543. http://dx.doi.org/10.3390/su11236543.
Guarda, P., Galilea, P., Handy, S., Muñoz, J.C., Ortúzar, J.d.D., 2016a. Decreasing fare evasion without fines? A microeconomic analysis. Res. Transp. Econ. 59, 151–158. http://dx.doi.org/10.1016/j.retrec.2016.06.001.
Guarda, P., Galilea, P., Paget-Seekins, L., Ortúzar, J.d.D., 2016b. What is behind fare evasion in urban bus systems? An econometric approach. Transp. Res. A 84, 55–71. http://dx.doi.org/10.1016/j.tra.2015.10.008.
Guzman, L.A., Arellana, J., Camargo, J.P., 2021. A hybrid discrete choice model to understand the effect of public policy on fare evasion discouragement in Bogotá's bus rapid transit. Transp. Res. A 151, 140–153. http://dx.doi.org/10.1016/j.tra.2021.07.009.
Harrison, G.W., Rutherford, T.F., Tarr, D.G., 2002. Trade policy options for Chile: The importance of market access. World Bank Econ. Rev. 16, 49–79. http://dx.doi.org/10.1093/wber/16.1.49.
Hörcher, D., Borger, B.D., Seifu, W., Graham, D.J., 2020. Public transport provision under agglomeration economies. Reg. Sci. Urban Econ. 81, http://dx.doi.org/10.1016/j.regsciurbeco.2019.103503.

Hörcher, D., Graham, D.J., 2020. Maas economics: Should we fight car ownership with subscriptions to alternative modes? Econ. Transp. 22, 100167. http://dx.doi.org/10.1016/j.ecotra.2020.100167.

Hörcher, D., Graham, D.J., Anderson, R.J., 2018. The economic inefficiency of travel passes under crowding externalities and endogenous capacity. J. Transp. Econ. Policy 52, 1–22, URL: https://www.ingentaconnect.com/contentone/lse/jtep/2018/00000052/00000001/art00002.

Hörcher, D., Tirachini, A., 2021. A review of public transport economics. Econ. Transp. 25, 100196. http://dx.doi.org/10.1016/j.ecotra.2021.100196.

Jansson, J.O., 1980. Simple bus line model for optimisation of service frequency and bus size. J. Transp. Econ. Policy 14, 53–80.

Jansson, J.O., 1984. Transport System Optimization and Pricing. In: Monograph, Wiley Chichester.

Jara-Díaz, S., 2007. Transport Economic Theory. Elsevier Ltd, http://dx.doi.org/10.1108/9780080548999.

Jara-Díaz, S., Fielbaum, A., Gschwender, A., 2020. Strategies for transit fleet design considering peak and off-peak periods using the single-line model. Transp. Res. B 142, 1–18. http://dx.doi.org/10.1016/j.trb.2020.09.012.

Jara-Díaz, S., Gschwender, A., 2003. Towards a general microeconomic model for the operation of public transport. Transp. Rev. 23, 453–469. http://dx.doi.org/10.1080/0144164032000048922.

Jara-Díaz, S., Gschwender, A., 2005. Making pricing work in public transport provision. In: Button, K., Hensher, D.A. (Eds.), Handbook of Transport Strategy, Policy and Institutions. Elsevier Ltd, Amsterdam, pp. 447–459. http://dx.doi.org/10.1108/9780080456041-026.

Jara-Díaz, S., Gschwender, A., 2009. The effect of financial constraints on the optimal design of public transport services. Transportation 36, 65–75. http://dx.doi.org/10.1007/s11116-008-9182-8.

Jara-Díaz, S., Gschwender, A., Hörcher, D., 2022. Public transport: design, scale and pricing. In: Tirachini, A., Hörcher, D., Verhoef, E.T. (Eds.), Handbook on Transport Pricing and Financing. Edward Elgar Publishing Ltd., pp. 171–189.

Killias, M., Scheidegger, D., Nordenson, P., 2009. The effects of increasing the certainty of punishment: A field experiment on public transportation. Eur. J. Criminol. 6, 387–400. http://dx.doi.org/10.1177/1477370809337881.

Kooreman, P., 1993. Fare evasion as a result of expected utility maximisation: Some empirical support. J. Transp. Econ. Policy 27, 69–74.

Kraus, M., 2003. A new look at the two-mode problem. J. Urban Econ. 54, 511–530. http://dx.doi.org/10.1016/S0094-1190(03)00079-2.

Laboratiorio de Innovacion Pública, 2018. Informe Final Laboratorio 502: Estudio exploratorio sobre las causas y posibles soluciones de la evasión en Transantiago. Technical Report, Laboratorio de innovación pública, Santiago, URL: https://www.economist.com/graphic-detail/2020/10/10/faith-in-government-declines-when-mobile-internet-arrives.

Laffont, J.J., Tirole, J., 1993. A Theory of Incentives in Procurement and Regulation. The MIT Press, Cambridge, Massachusetts.

Lee, J., 2011. Uncovering San Francisco, California, muni's proof-of-payment patterns to help reduce fare evasion. Transp. Res. Rec. 75–84. http://dx.doi.org/10.3141/2216-09.

Manríquez, M., 2019. Determinación del número óptimo de fiscalizadores en buses del Transantiago para hacer frente a la evasión. In: Congreso Chileno de ingenenierı́a de transporte.

Matas, A., Raymond, J.L., Ruiz, A., 2020. Economic and distributional effects of different fare schemes: Evidence from the metropolitan region of Barcelona. Transp. Res. A 138, 1–14. http://dx.doi.org/10.1016/j.tra.2020.05.014.

Milioti, C., Panoutsopoulos, A., Kepaptsoglou, K., Tyrinopoulos, Y., 2020. Key drivers of fare evasion in a metro system: Evidence from Athens, Greece. Case Stud. Transp. Policy 8, 778–783. http://dx.doi.org/10.1016/j.cstp.2020.06.005.

Mohring, H., 1972. American economic association optimization and scale economies in urban bus transportation. Source: Am. Econ. Rev. 62, 591–604, URL: http://www.jstor.org/stable/1806101%5Cnhttp://www.jstor.org/page/info/about/policies/terms.jsp.

Mohring, H., 1976. Transportation Economics. Ballinger, Cambridge, MA.

Ortúzar, J.d.D., Willumsen, L.G., 2011. Modelling Transport. Wiley, p. 586. http://dx.doi.org/10.1002/9781119993308.

Pestieau, P., Possen, U.M., 1991. Tax evasion and occupational choice. J. Public Econ. 45, 107–125. http://dx.doi.org/10.1016/0047-2727(91)90050-C.

Porath, K., Galilea, P., 2020. Temporal analysis of fare evasion in Transantiago: A socio-political view. Res. Transp. Econ. 83, http://dx.doi.org/10.1016/j.retrec.2020.100958.

Pressman, I., 1970. A mathematical formulation of the peak-load pricing problem. Source: Bell J. Econ. Manag. Sci. 1, 304–326.

Probogotá, 2022. Hablemos de evasión: Aproximación al fenómeno y problema en Transmilenio. Technical Report, Probogotá.

Proost, S., van Dender, K., 2008. Optimal urban transport pricing in the presence of congestion, economies of density and costly public funds. Transp. Res. A 42, 1220–1230. http://dx.doi.org/10.1016/j.tra.2008.03.009.

Ramírez, V., Galilea, P., Poblete, J., Silva, H.E., 2022. Team-based incentives in transportation firms: An experiment. Transp. Res. A 164, 1–12. http://dx.doi.org/10.1016/j.tra.2022.07.012.

Ramos, R., Silva, H., 2022. Public transport subsidies and the shadow cost of public funds. SSRN Electron. J. http://dx.doi.org/10.2139/ssrn.4095922.

Salis, S., Barabino, B., Useli, B., 2018. Segmenting fare evader groups by factor and cluster analysis. In: WIT Transactions on the Built Environment, Vol. 176. WIT Press, pp. 503–515. http://dx.doi.org/10.2495/UT170431.

Sandmo, A., 1981. Income tax evasion, labour supply, and the equity—efficiency tradeoff. J. Public Econ. 16, 265–288. http://dx.doi.org/10.1016/0047-2727(81)90001-3.

Sasaki, Y., 2014. Optimal choices of fare collection systems for public transportations: Barrier versus barrier-free. Transp. Res. B 60, 107–114. http://dx.doi.org/10.1016/j.trb.2013.12.005.

Silva, H.E., 2021. The Mohring effect. In: International Encyclopedia of Transportation. pp. 263–266. http://dx.doi.org/10.1016/b978-0-08-102671-7.10050-8.

Silva, E.C., Kahn, C.M., 1993. Exclusion and moral hazard. The case of identical demand. J. Public Econ. 52, 217–235. http://dx.doi.org/10.1016/0047-2727(93)90021-K.

Small, K.A., Rosen, H.S., 1981. Applied welfare economics with discrete choice models. Econometrica 49, 105. http://dx.doi.org/10.2307/1911129.

Small, K.A., Verhoef, E.T., 2007. The Economics of Urban Transportation. Routledge, London, pp. 1–276. http://dx.doi.org/10.4324/9780203642306.

Sun, Y., Guo, Q., Schonfeld, P., Li, Z., 2016. Implications of the cost of public funds in public transit subsidization and regulation. Transp. Res. A 91, 236–250.

Sun, S., Szeto, W.Y., 2019. Optimal sectional fare and frequency settings for transit networks with elastic demand. Transp. Res. B 127, 147–177. http://dx.doi.org/10.1016/j.trb.2019.06.011.

Suquet, J.B., 2010. Drawing the line: how inspectors enact deviant behaviors. J. Serv. Mark. 24, 468–475, URL: https://hal-upec-upem.archives-ouvertes.fr/hal-01133097.

Tamblay, S., Gschwender, A., Dragicevic, M., Muñoz, J.C., 2017. Risks involved in transit provision of bus contracts. Transp. Res. Rec. J. Transp. Res. Board 2649, 89–96. http://dx.doi.org/10.3141/2649-10.

Tirachini, A., Hensher, D.A., 2011. Bus congestion, optimal infrastructure investment and the choice of a fare collection system in dedicated bus corridors. Transp. Res. B 45, 828–844. http://dx.doi.org/10.1016/j.trb.2011.02.006.

Tirachini, A., Hensher, D.A., 2012. Multimodal transport pricing: First best, second best and extensions to non-motorized transport. Transp. Rev. 32, 181–202. http://dx.doi.org/10.1080/01441647.2011.635318.

Tirachini, A., Hensher, D.A., Rose, J.M., 2014. Multimodal pricing and optimal design of urban public transport: The interplay between traffic congestion and bus crowding. Transp. Res. B 61, 33–54. http://dx.doi.org/10.1016/j.trb.2014.01.003.

Train, K.E., 2009. Discrete Choice Methods with Simulation, Vol. 9780521766, second ed. pp. 1–388. http://dx.doi.org/10.1017/CBO9780511805271.

Trandel, G.A., 1992. Evading the use tax on cross-border sales: Pricing and welfare effects. J. Public Econ. 49, 313–331. http://dx.doi.org/10.1016/0047-2727(92)90071-M.

Troncoso, R., de Grange, L., 2017. Fare evasion in public transport: A time series approach. Transp. Res. A 100, 311–318. http://dx.doi.org/10.1016/j.tra.2017.04.029.

Waters, J., 2015. Welfare implications of piracy with dynamic pricing and heterogeneous consumers. European J. Oper. Res. 240, 904–911. http://dx.doi.org/10.1016/j.ejor.2014.08.022.

Yap, M., Cats, O., van Arem, B., 2020. Crowding valuation in urban tram and bus transportation based on smart card data. Transp. A: Transp. Sci. 16, 23–42. http://dx.doi.org/10.1080/23249935.2018.1537319.

Yoon, K., 2002. The optimal level of copyright protection. Inf. Econ. Policy 14, 327–348. http://dx.doi.org/10.1016/S0167-6245(01)00062-2.

Zhang, J., Yang, H., Lindsey, R., Li, X., 2020. Modeling and managing congested transit service with heterogeneous users under monopoly. Transp. Res. B 132, 249–266. http://dx.doi.org/10.1016/j.trb.2019.04.012.

Zou, W., Mizokami, S., 2014. Incentive subsidy scheme design with elastic transport demand. J. Adv. Transp. 48, 927–941. http://dx.doi.org/10.1002/atr.1253.